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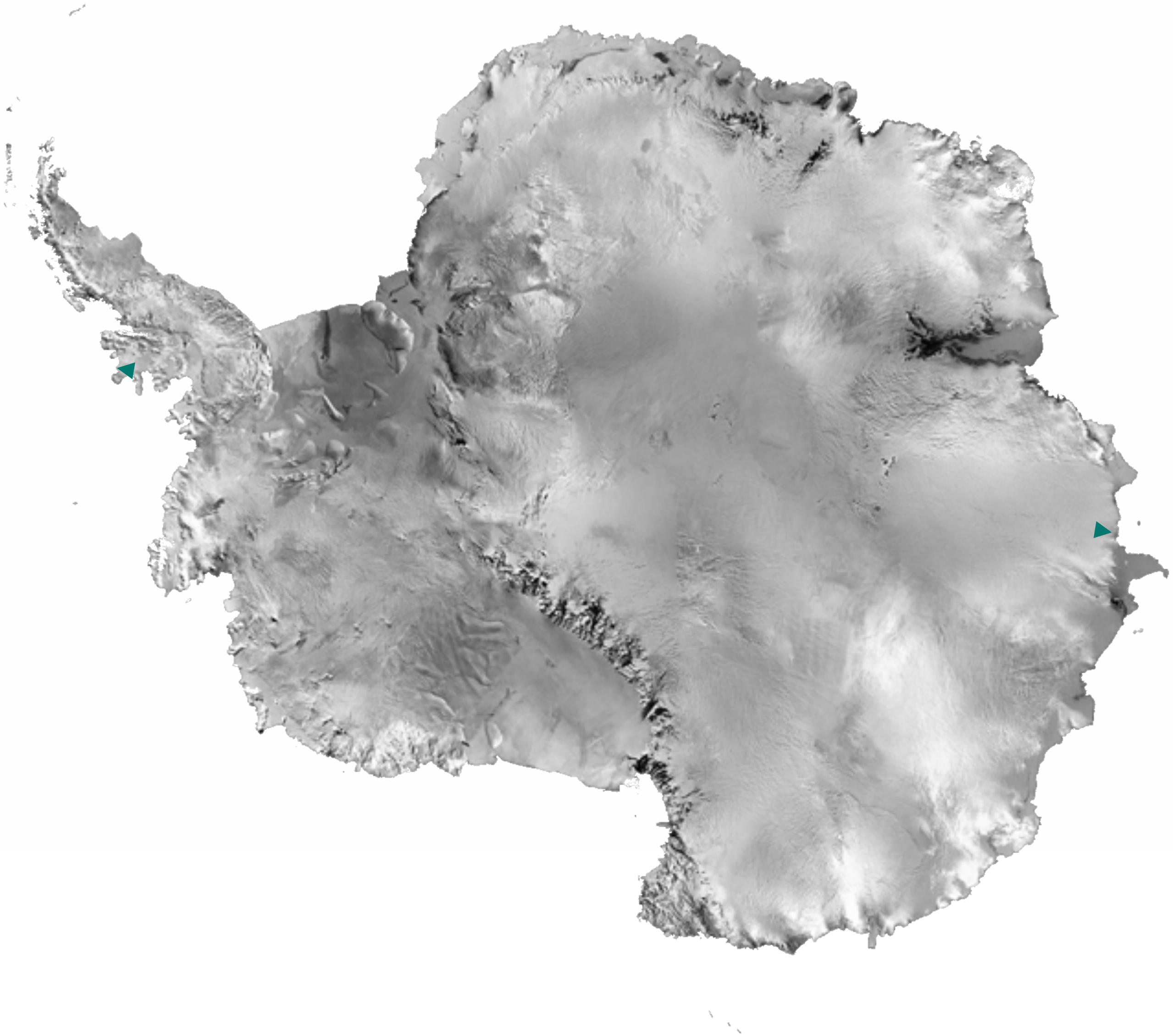
Von Anschauung und Meßdaten zur mathematischen Modellierung - Beispiel Glaziologie

20 March 2012

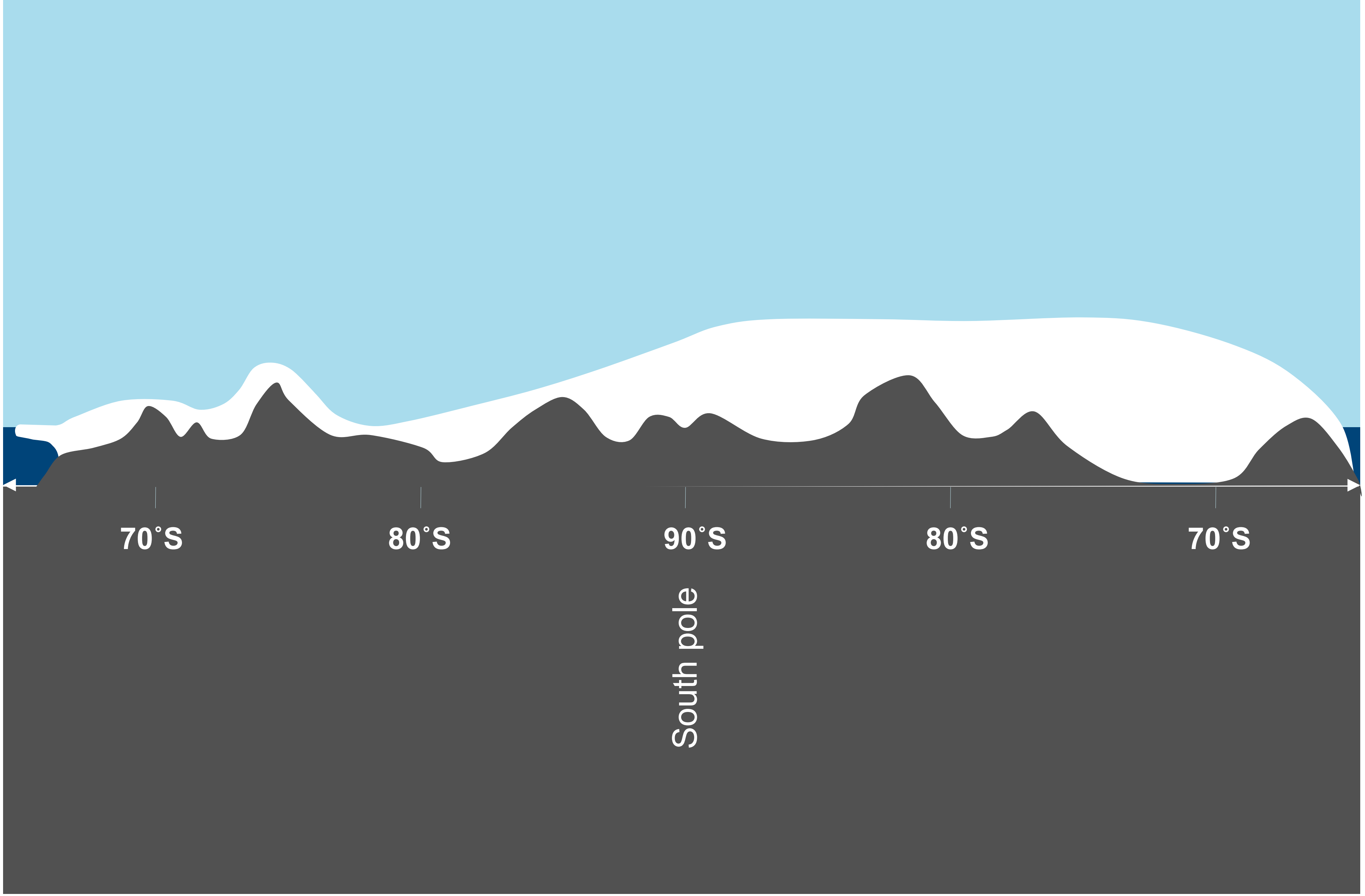
Angelika Humbert

Universität Hamburg

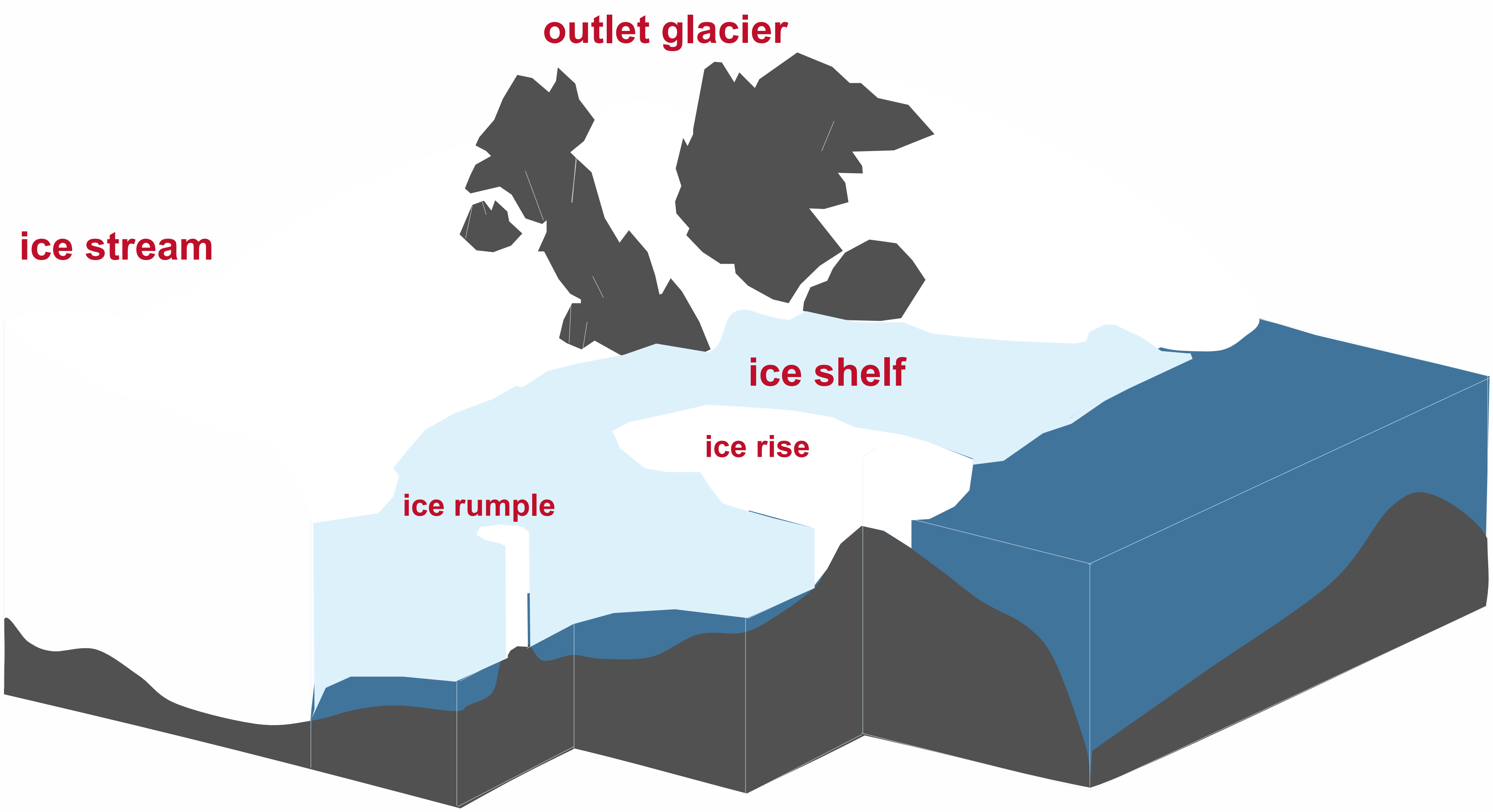
Ice sheets - Antarctica



Ice sheets - Antarctica



The system



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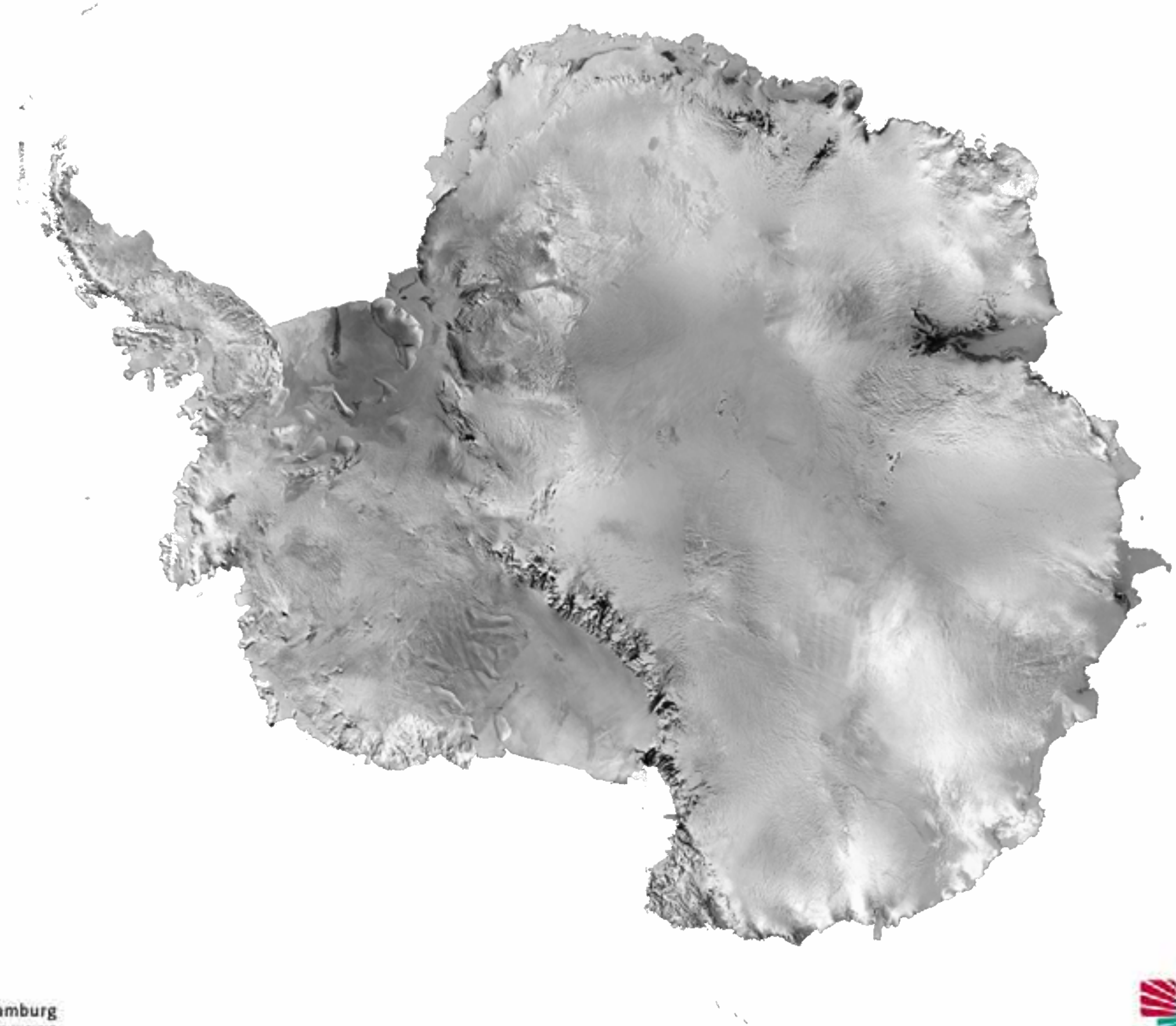
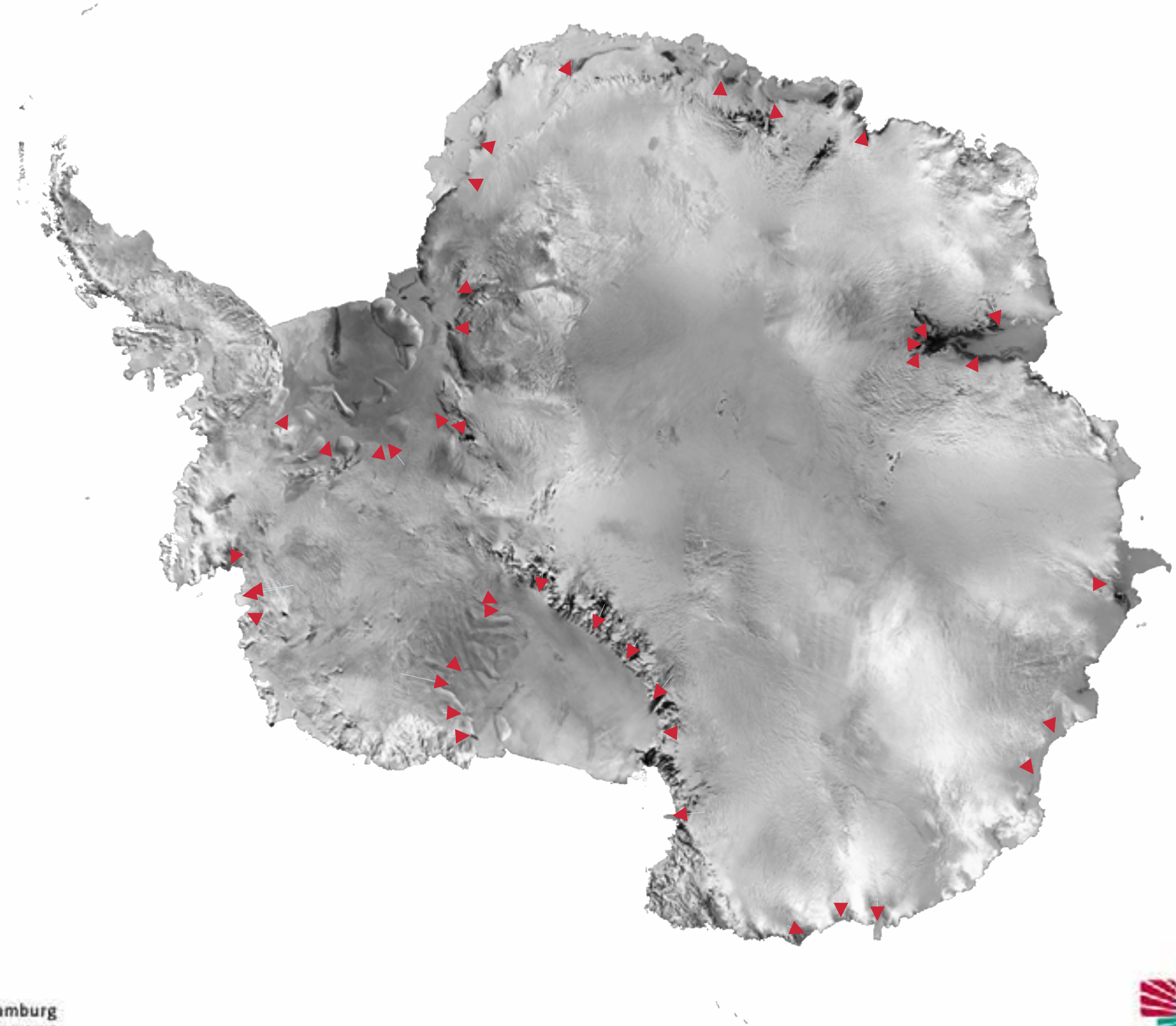
Ice sheets – Antarctica



after Rignot, 2006

Ice sheets - Antarctica

© USGS



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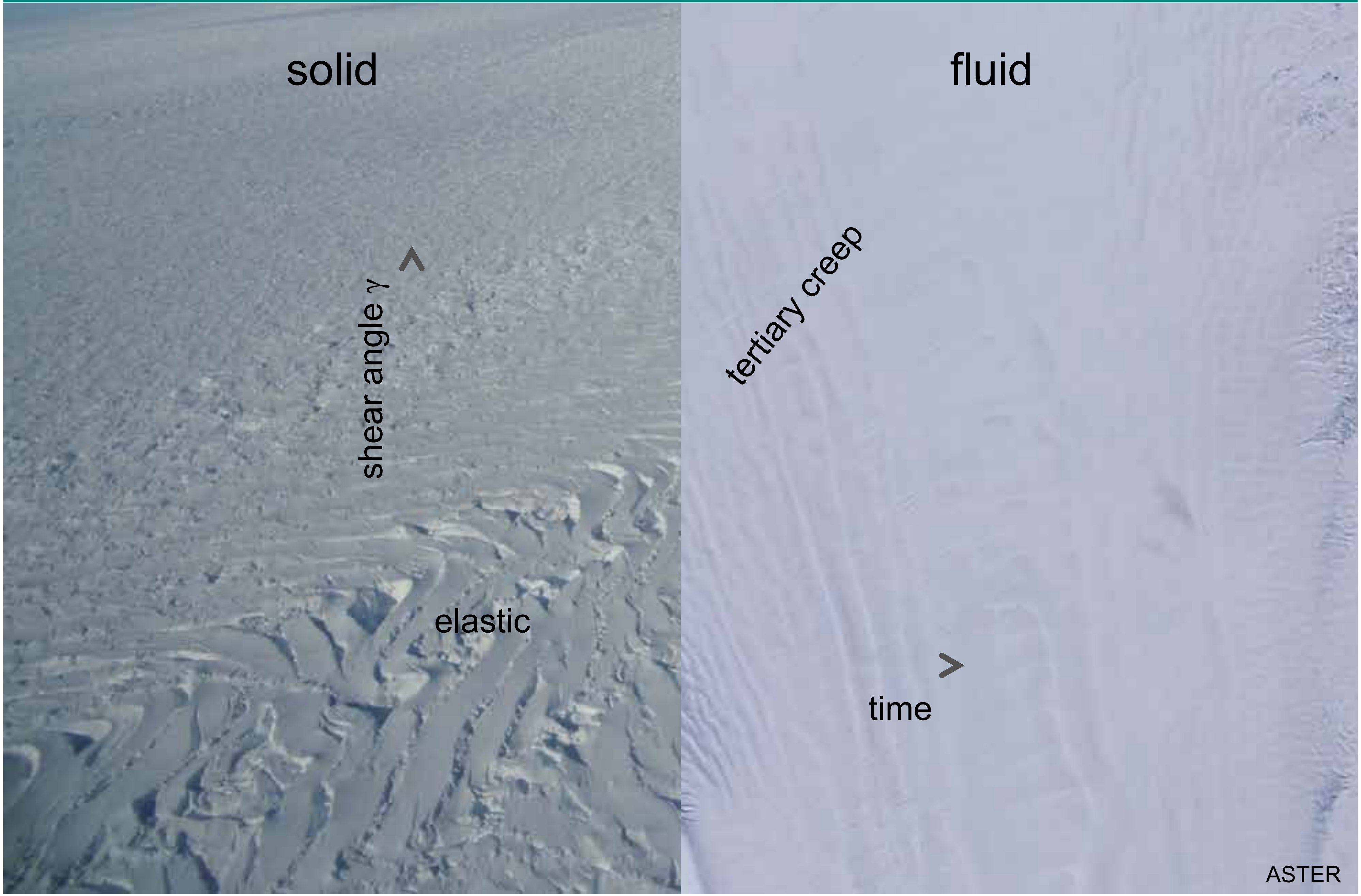
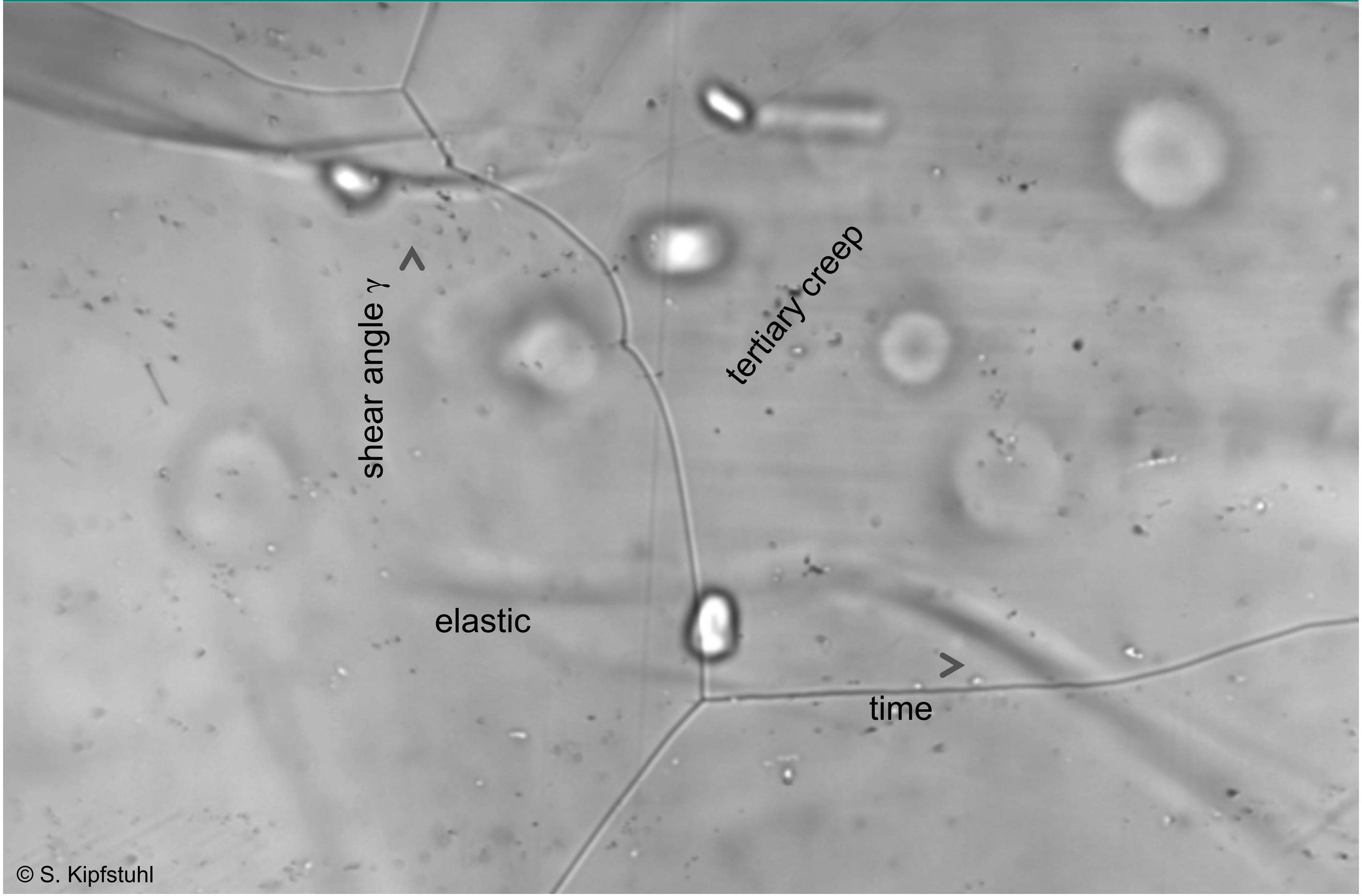
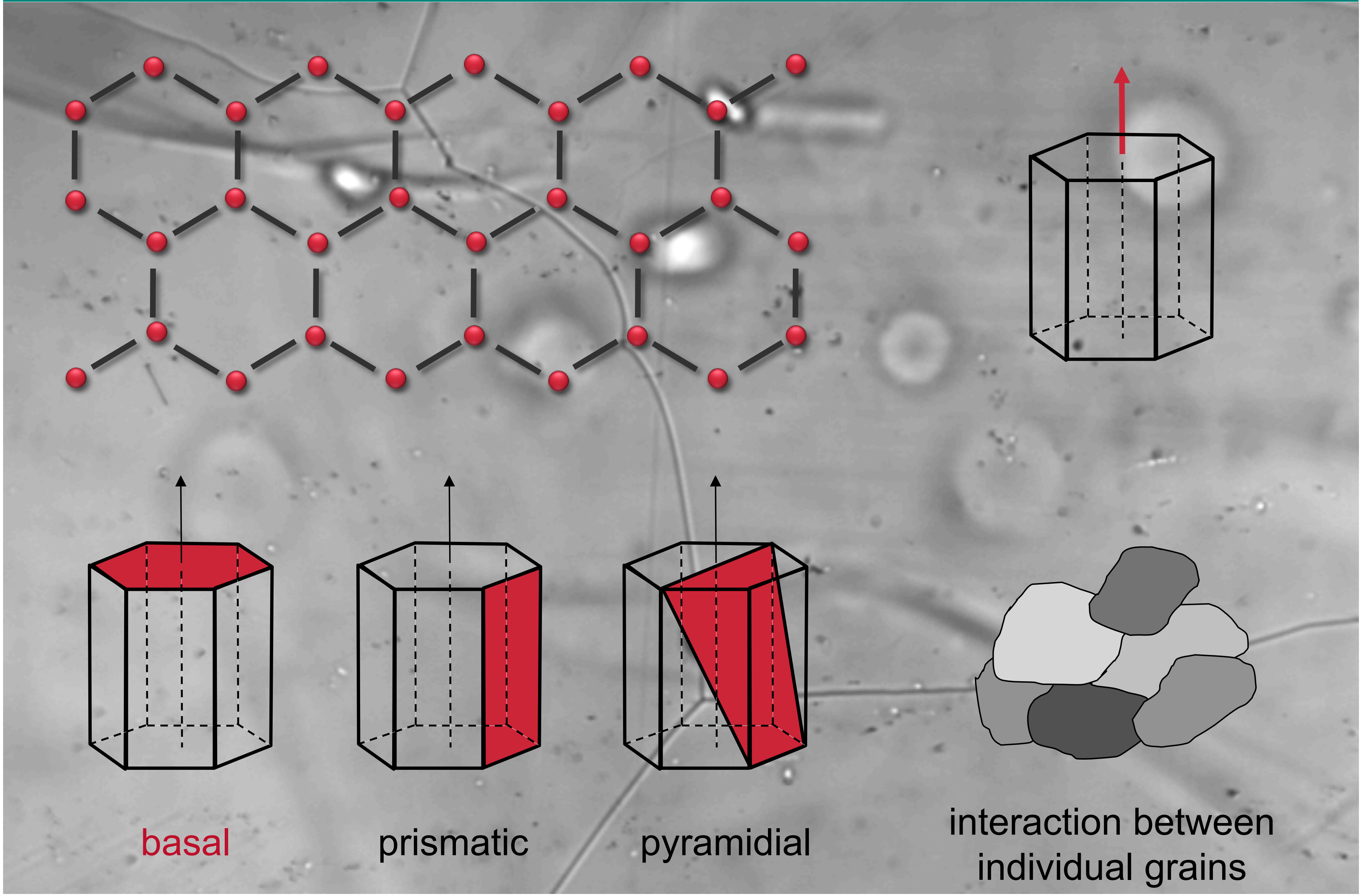
The nature of ice



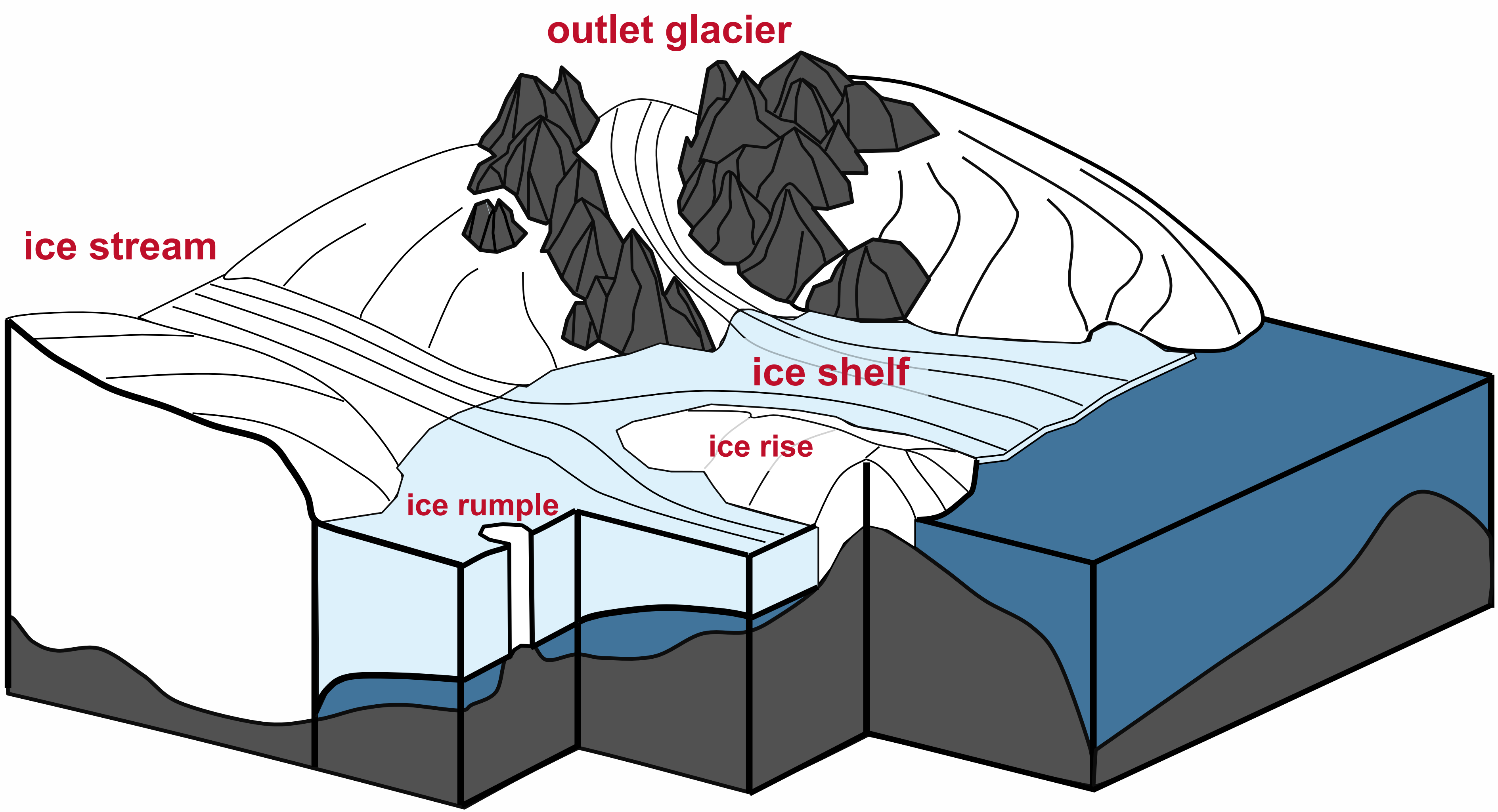
Deformation of polycrystalline ice



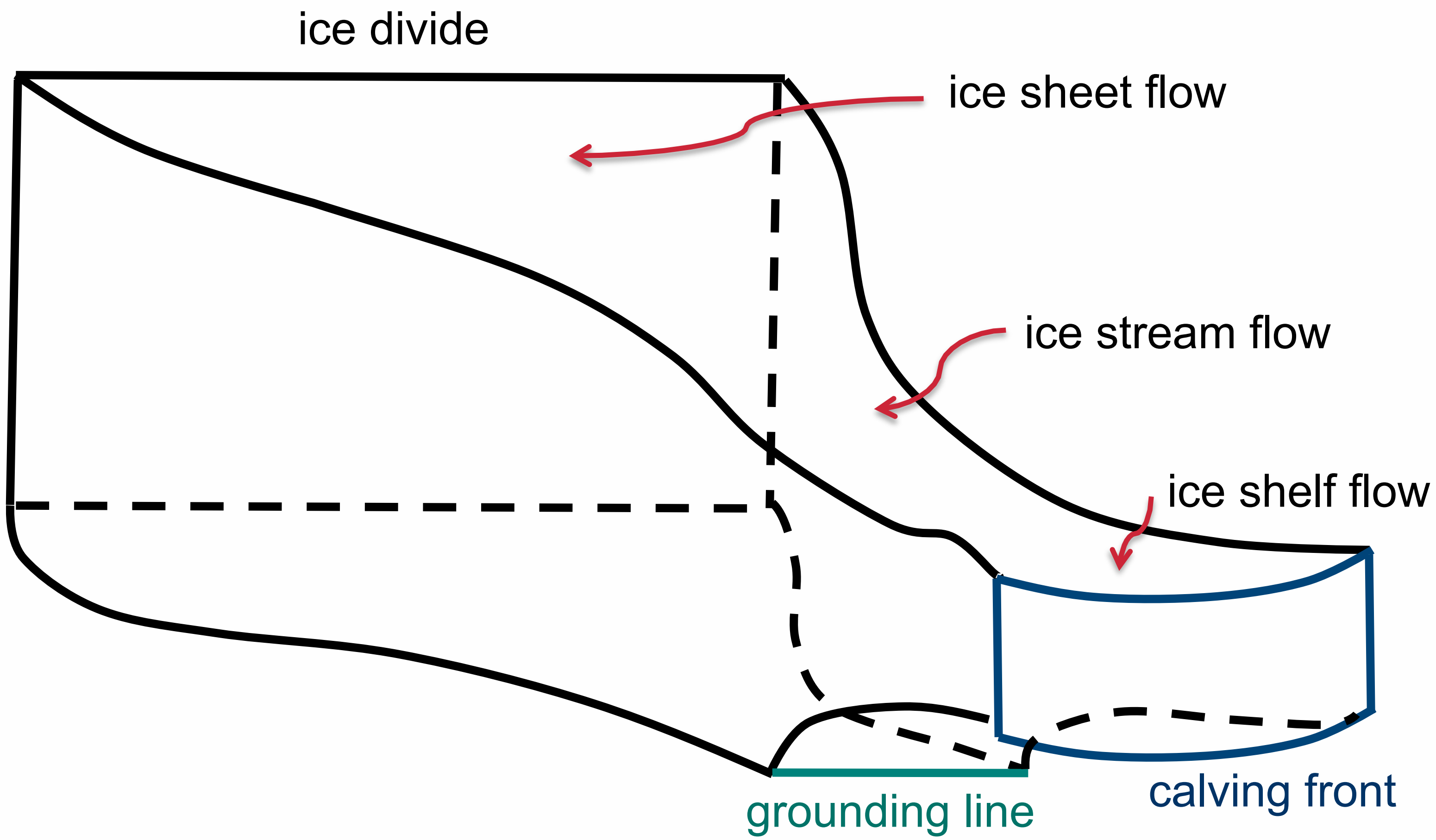
Physics of polycrystalline ice



The system



Stresses along a cross section of ice sheet 2 ice front



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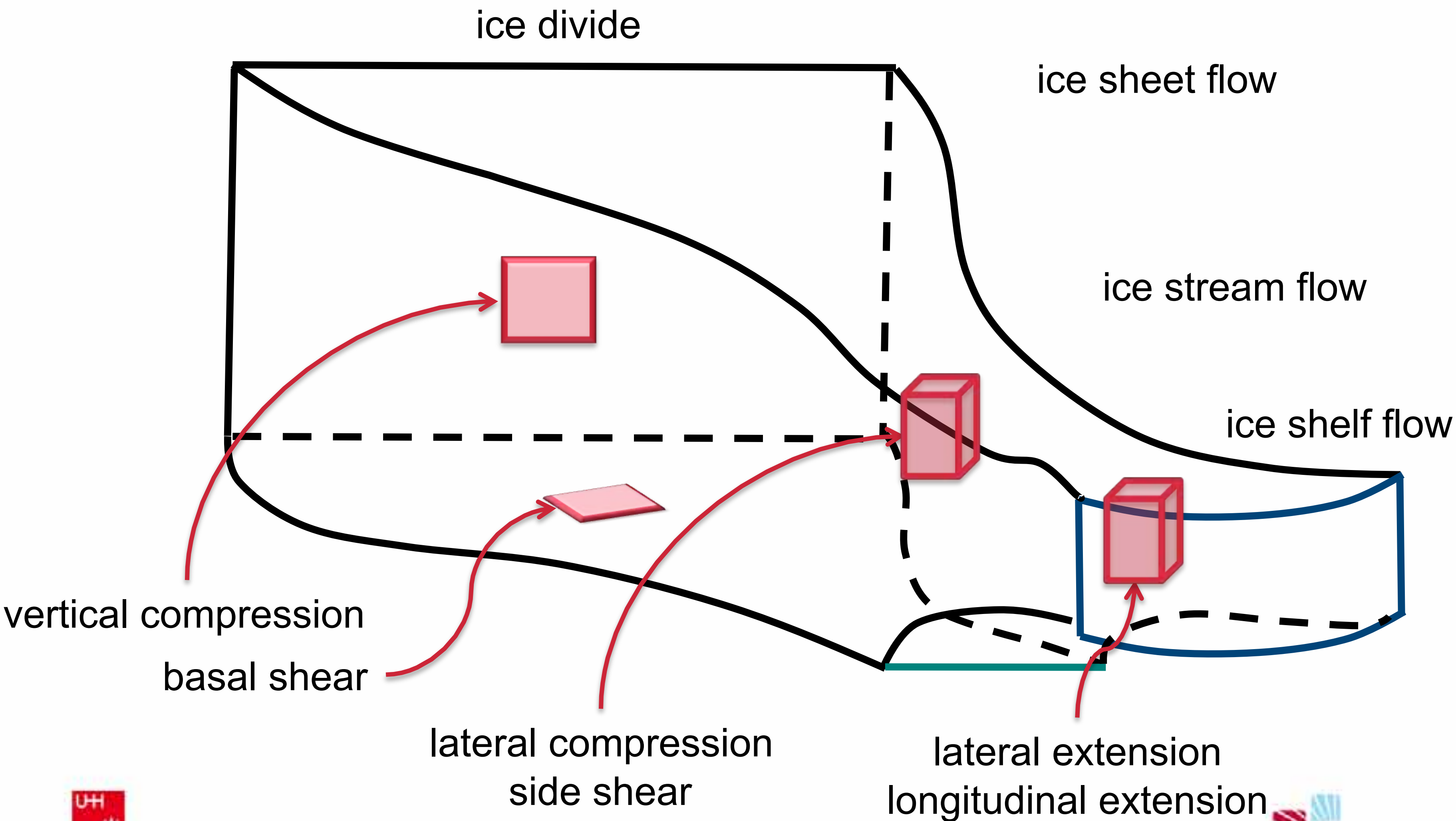
Stresses along a cross section of ice sheet 2 ice front



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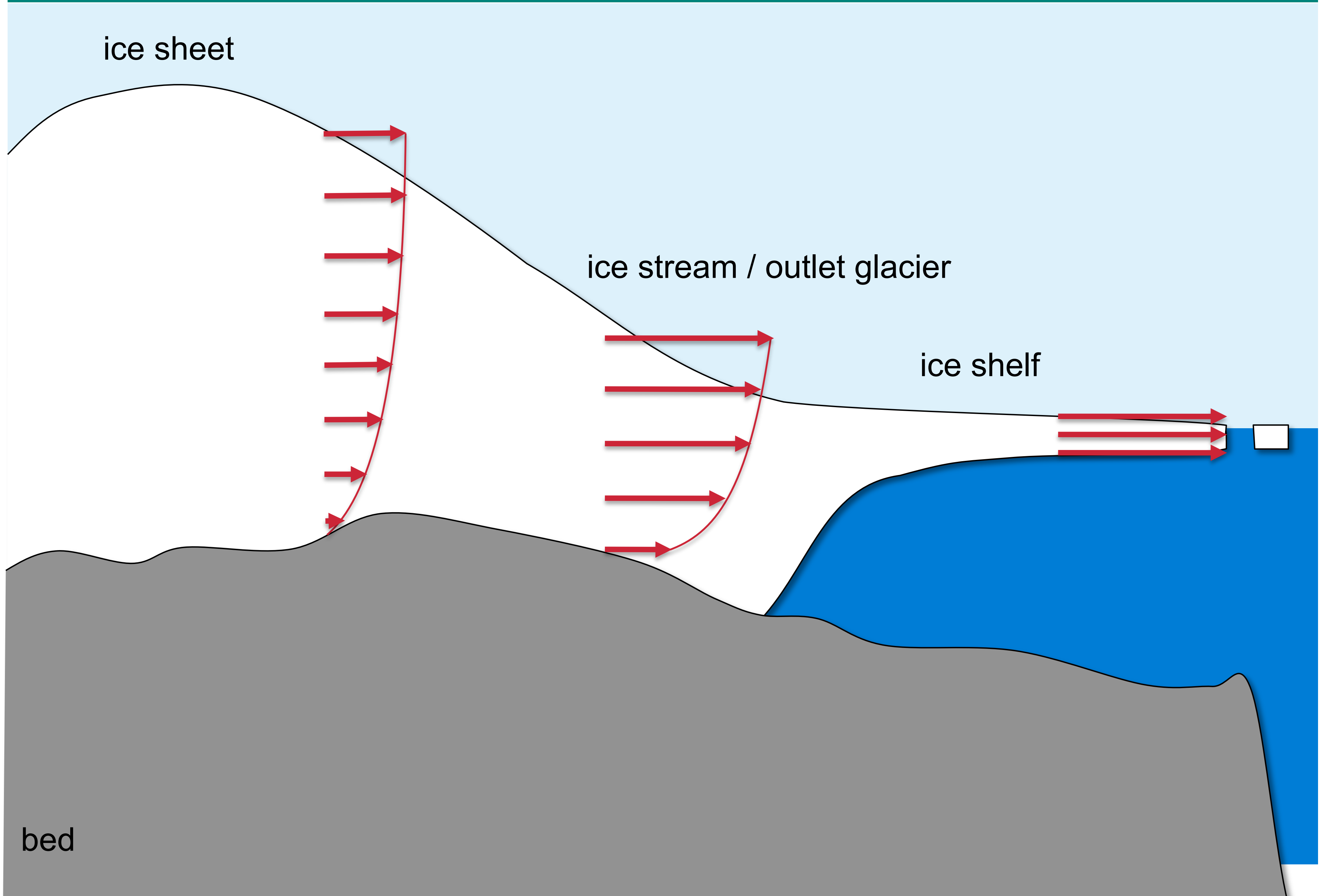
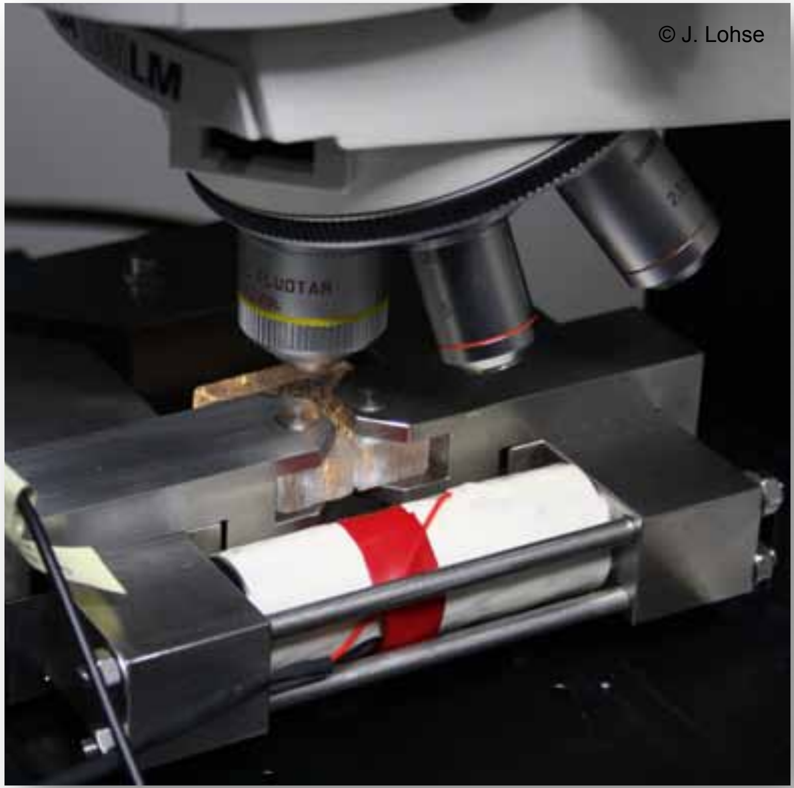
Horizontal velocity profiles



Observation versus experiment

observation



fracture mechanical experiment

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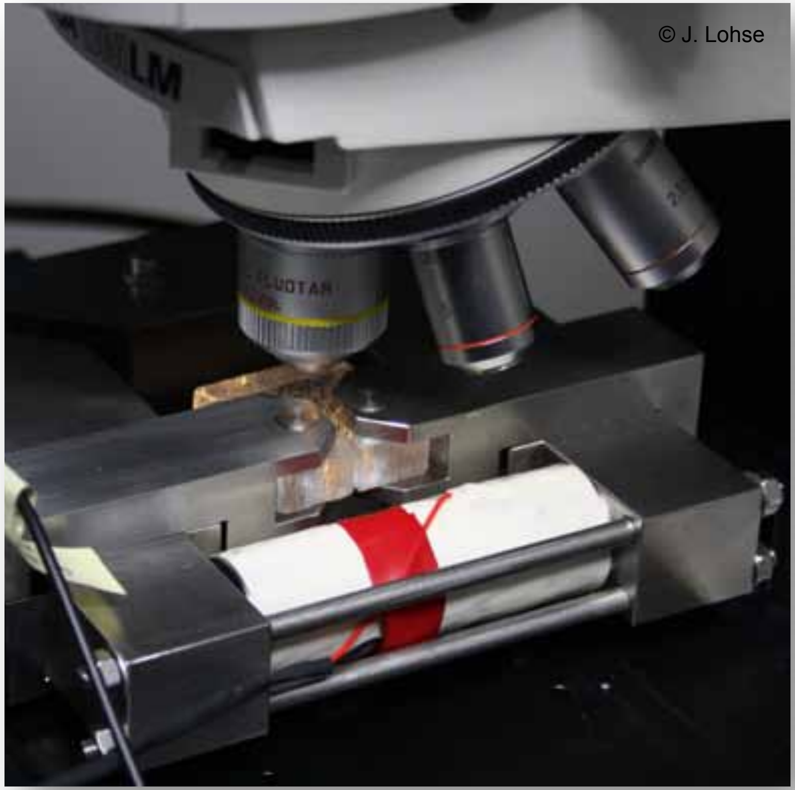
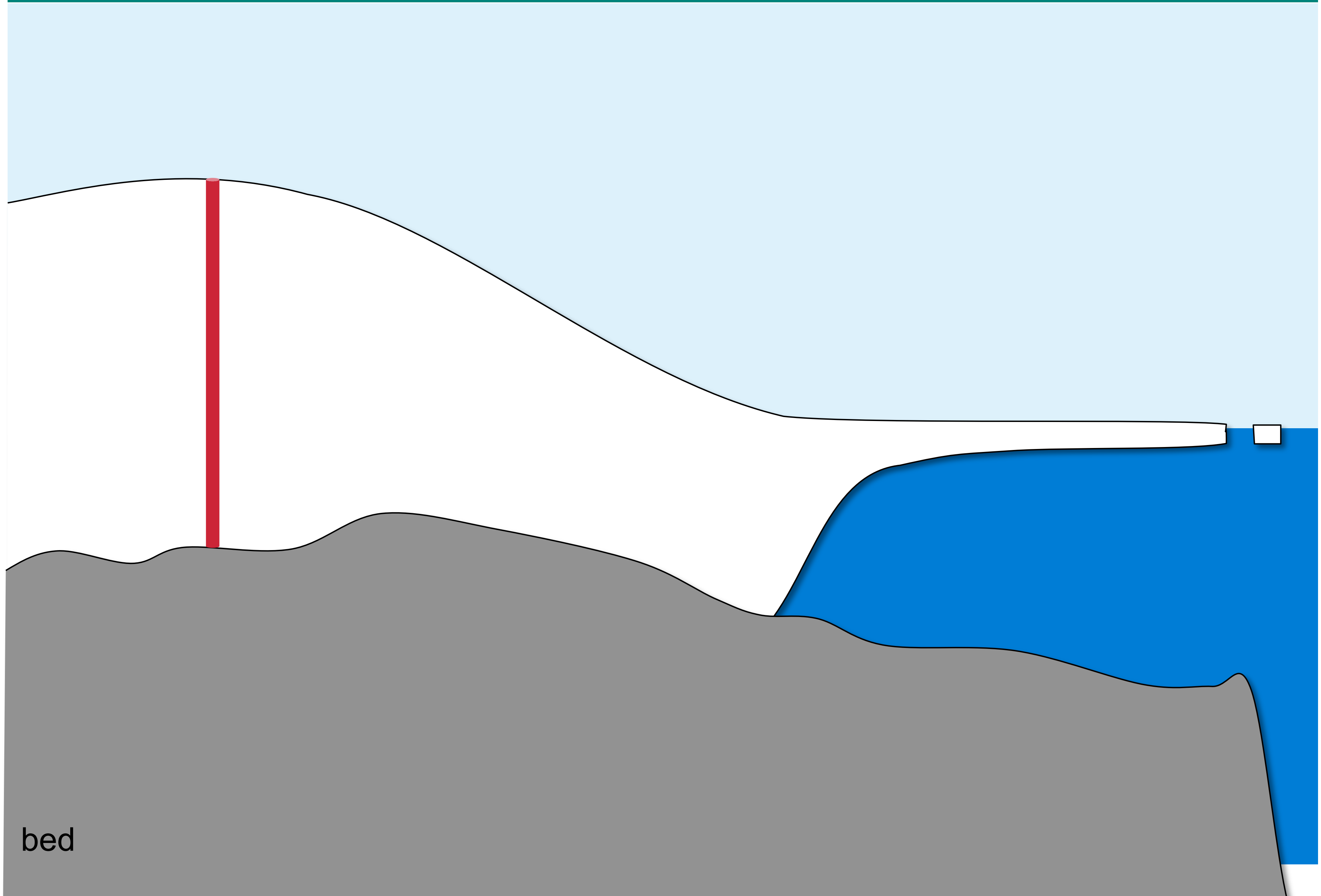
Observation versus experiment

observation

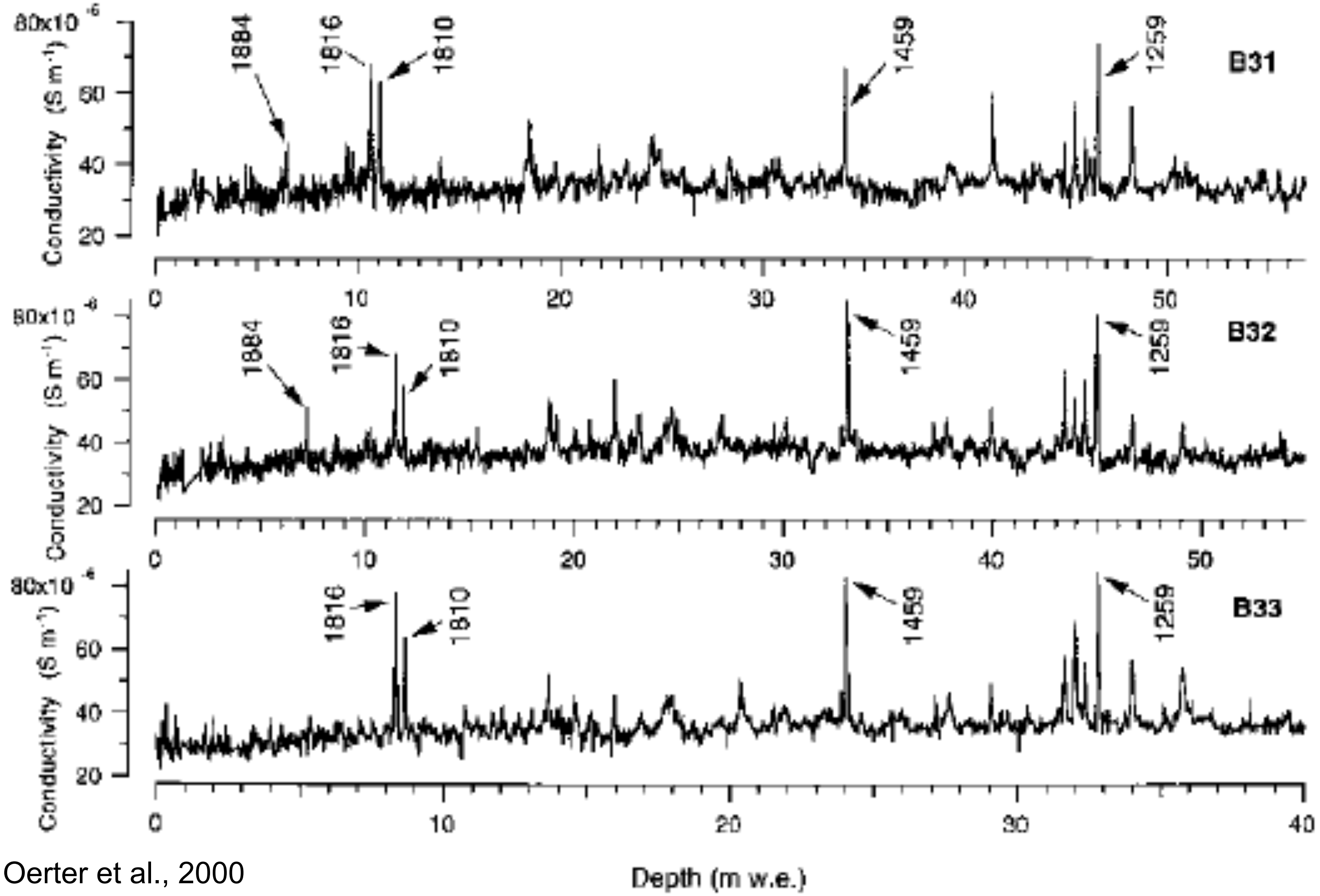


fracture mechanical experiment

Observational Methods

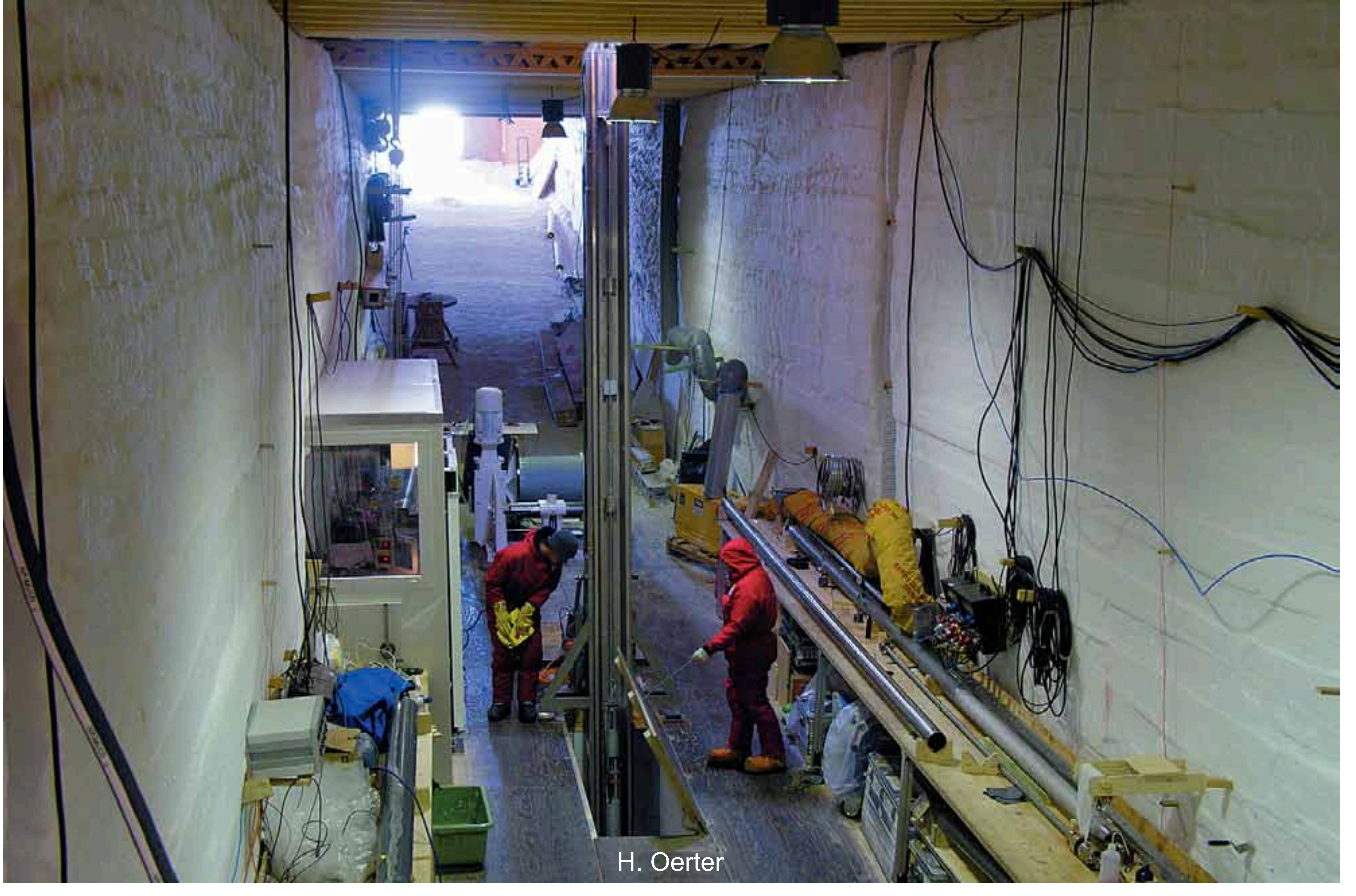


Shallow cores – accumulation rates

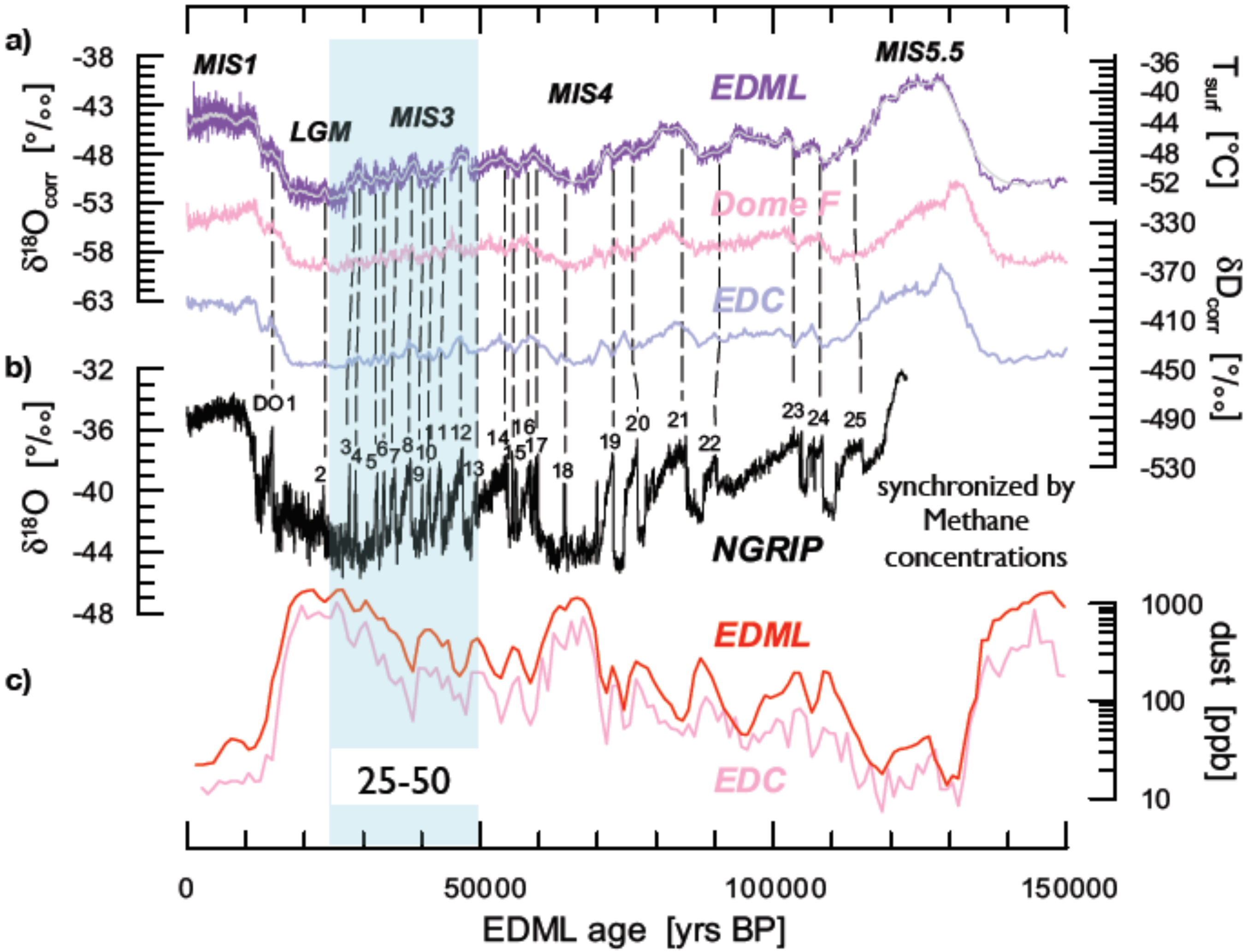


Oerter et al., 2000

Deep cores – climate history



Deep cores – climate history



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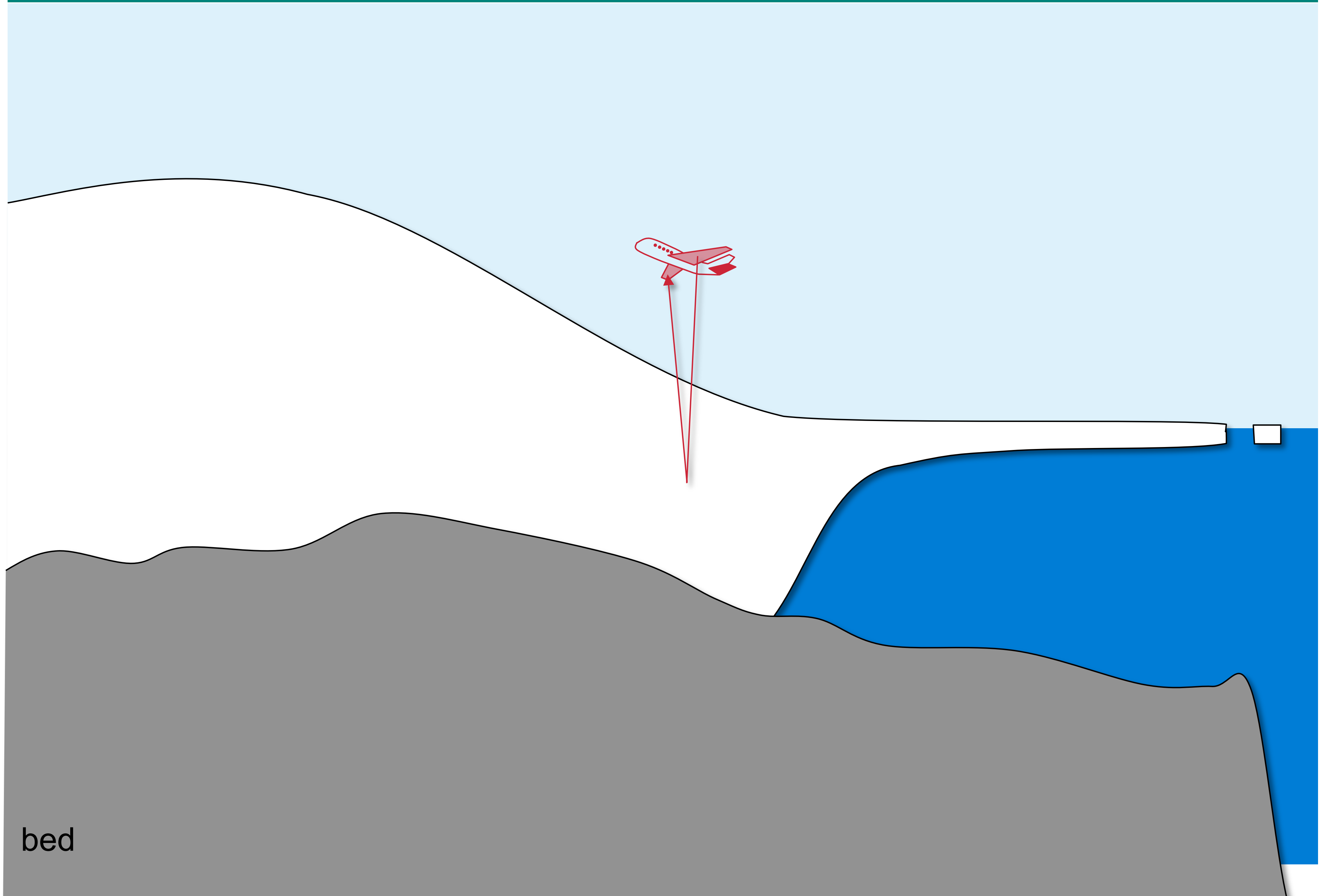
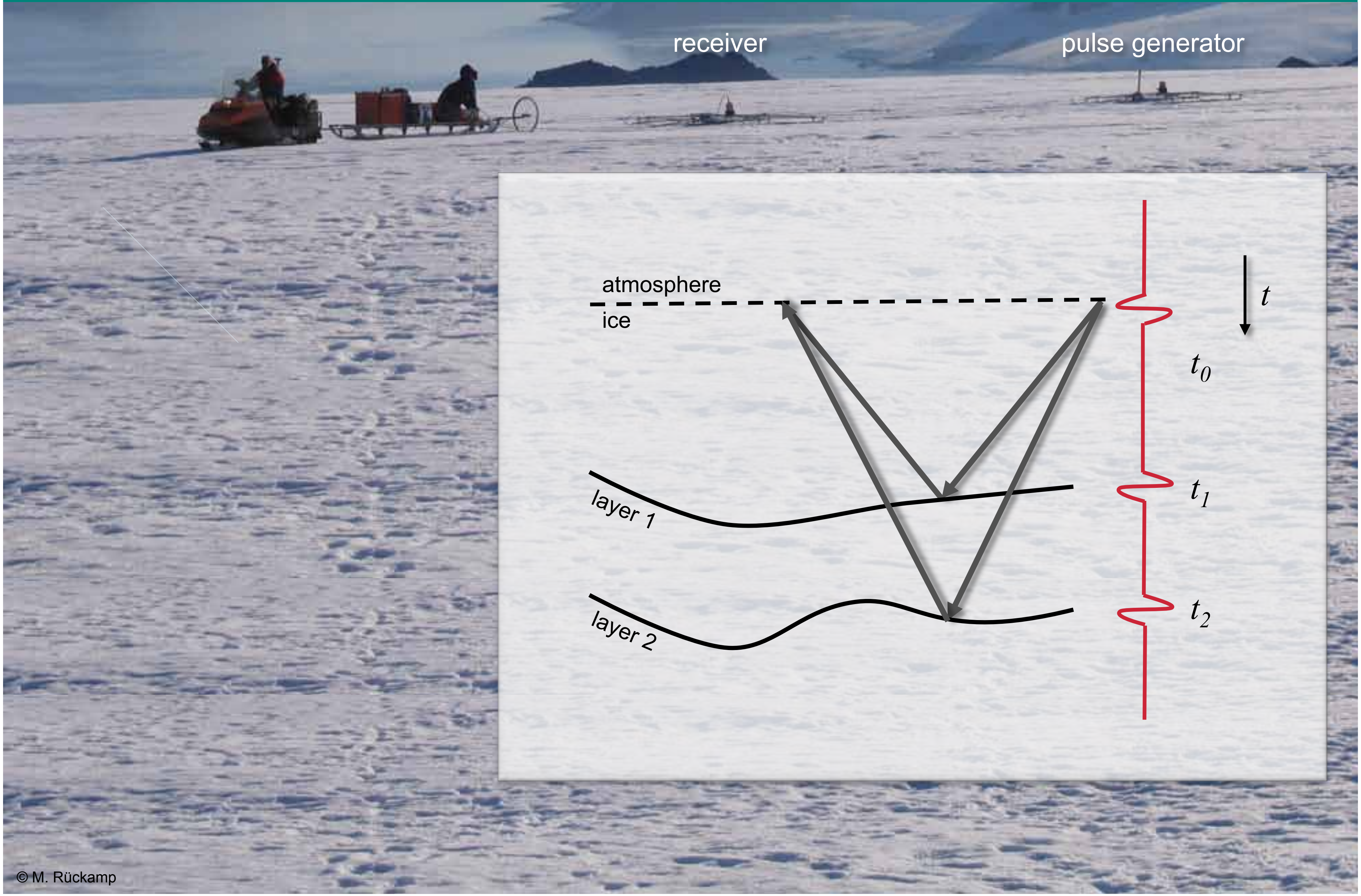
H. Oerter

Observational Methods



Radio echo sounding

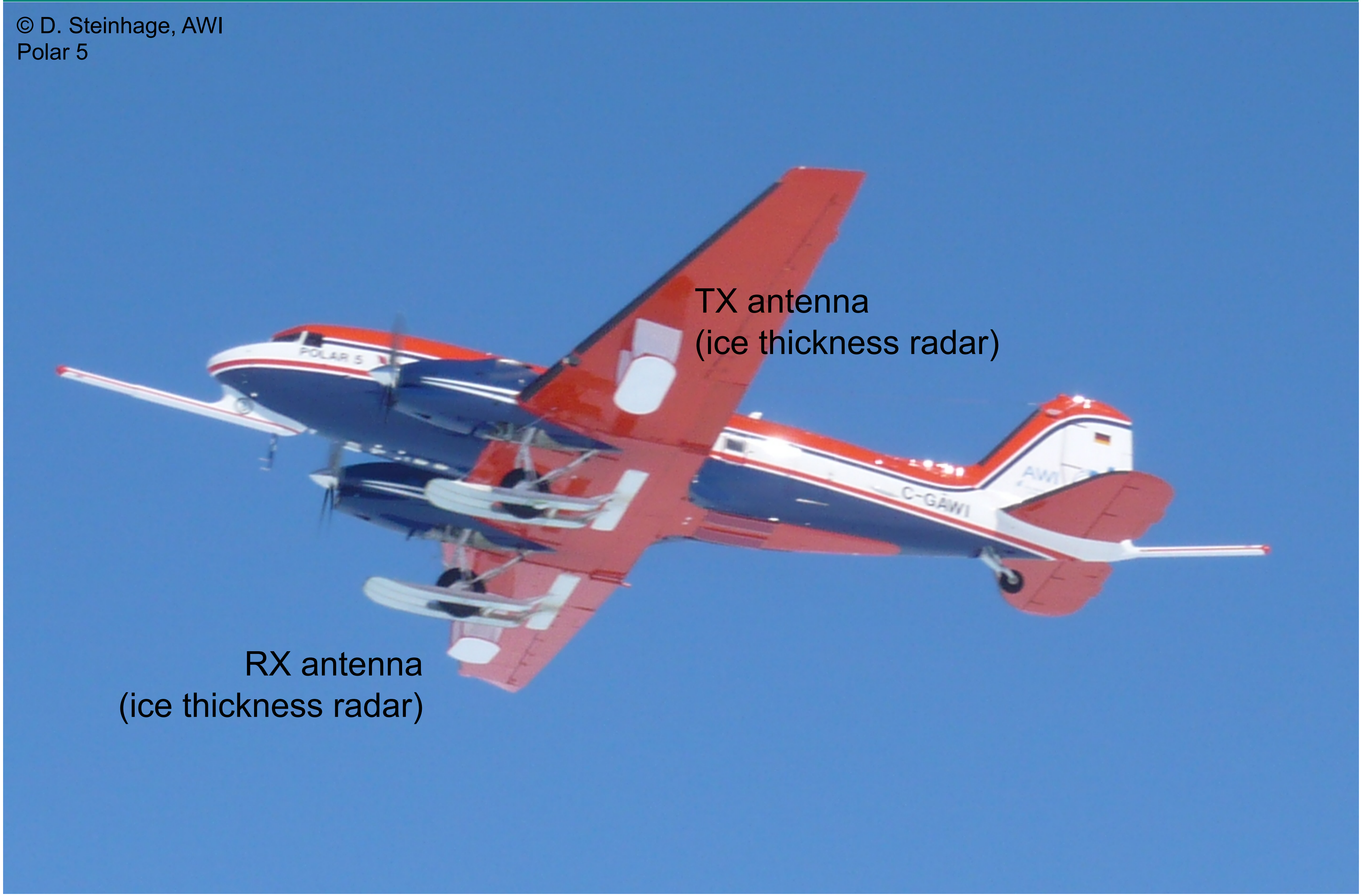
Radio echo sounding



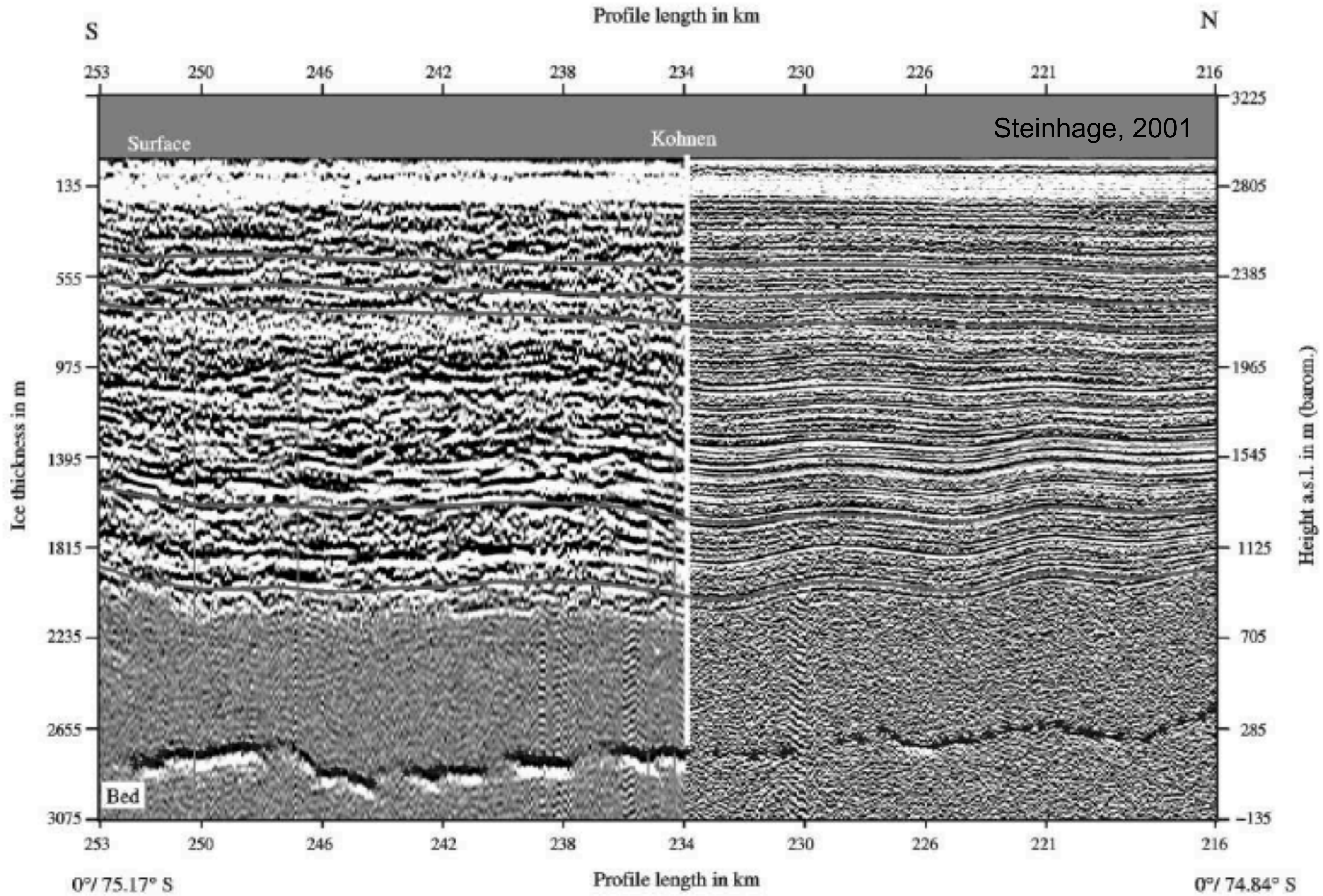
Radio echo sounding

Radio echo sounding

© D. Steinhage, AWI
Polar 5



Radio echo sounding



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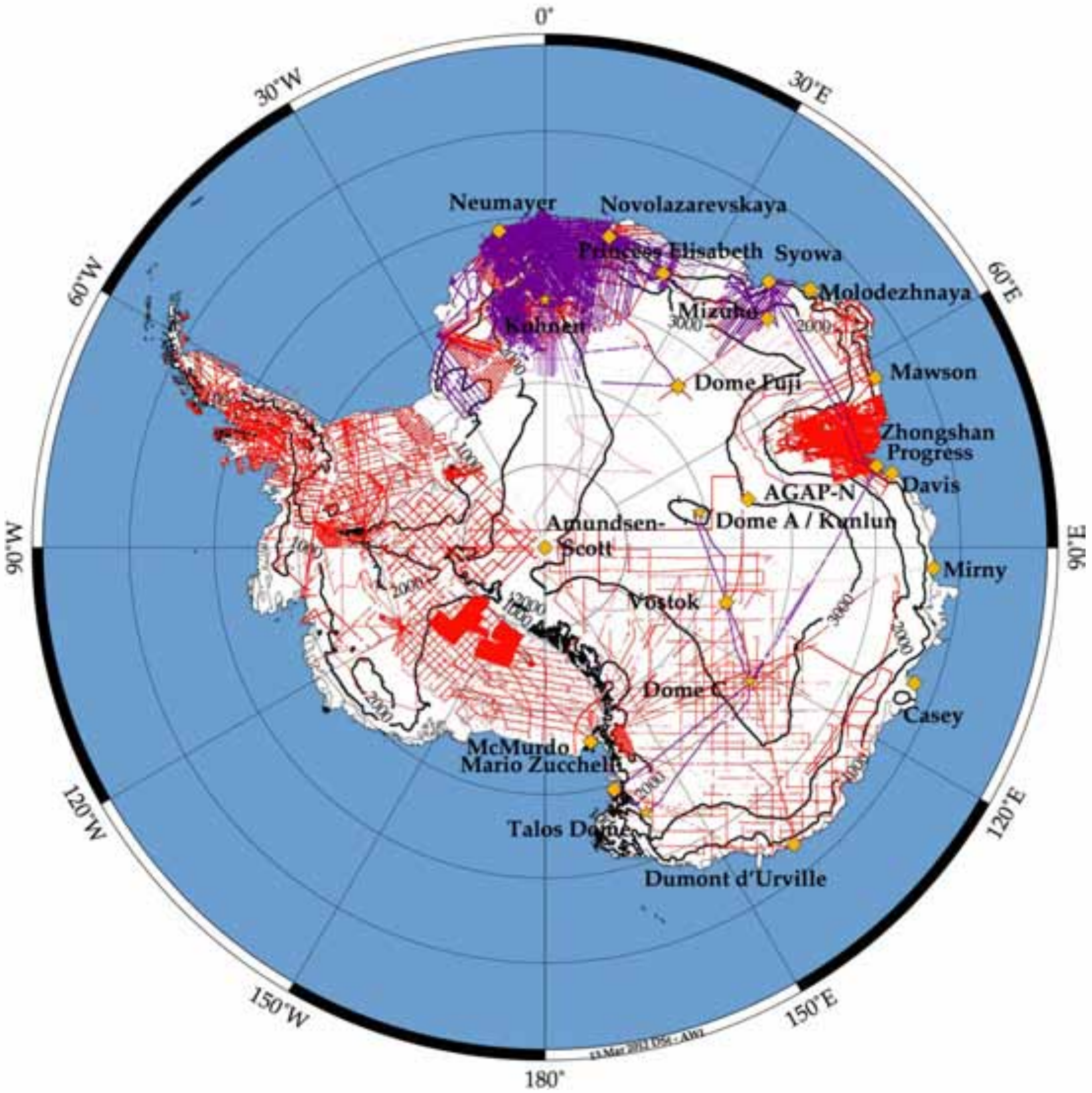
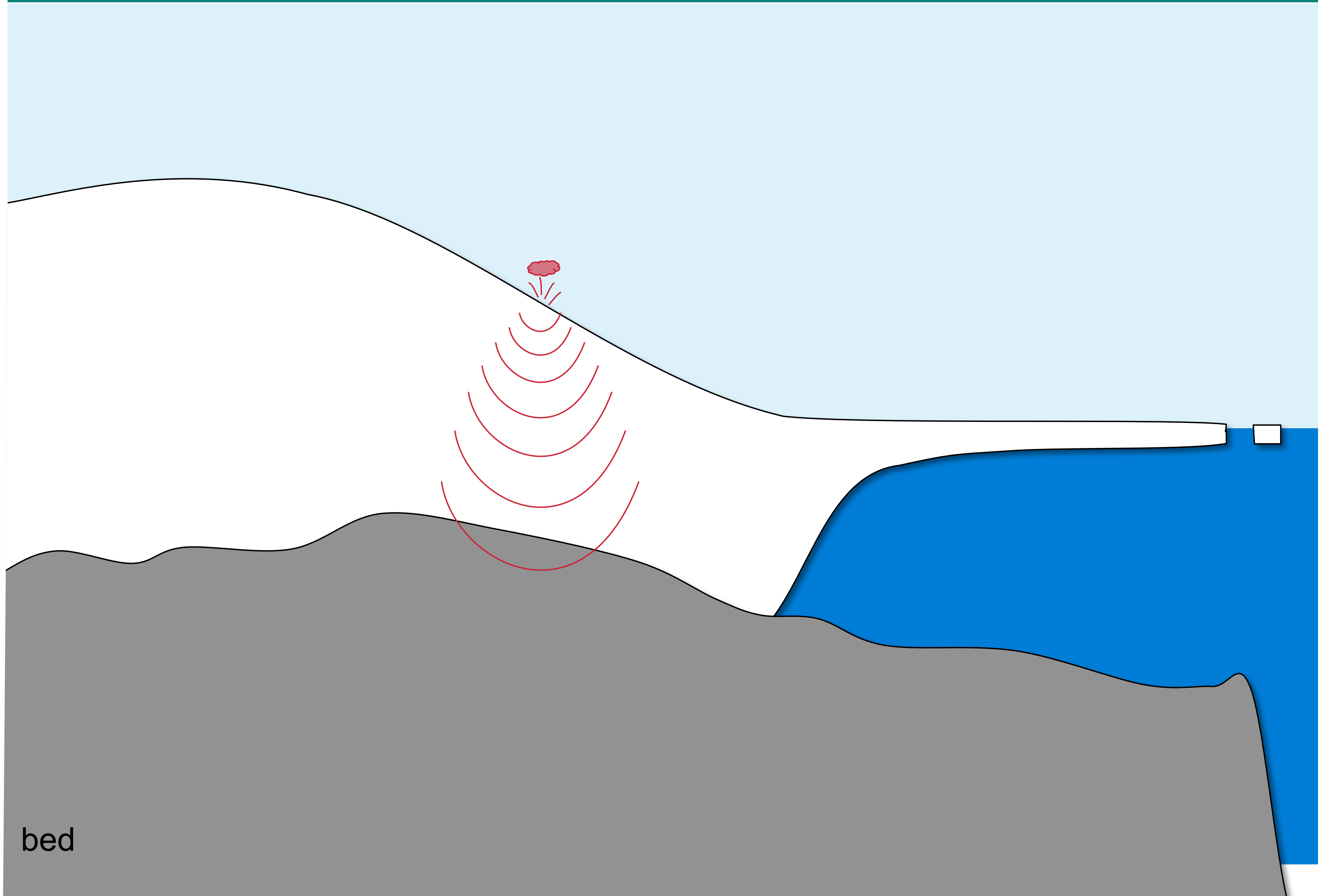
Data coverage – radio echo sounding

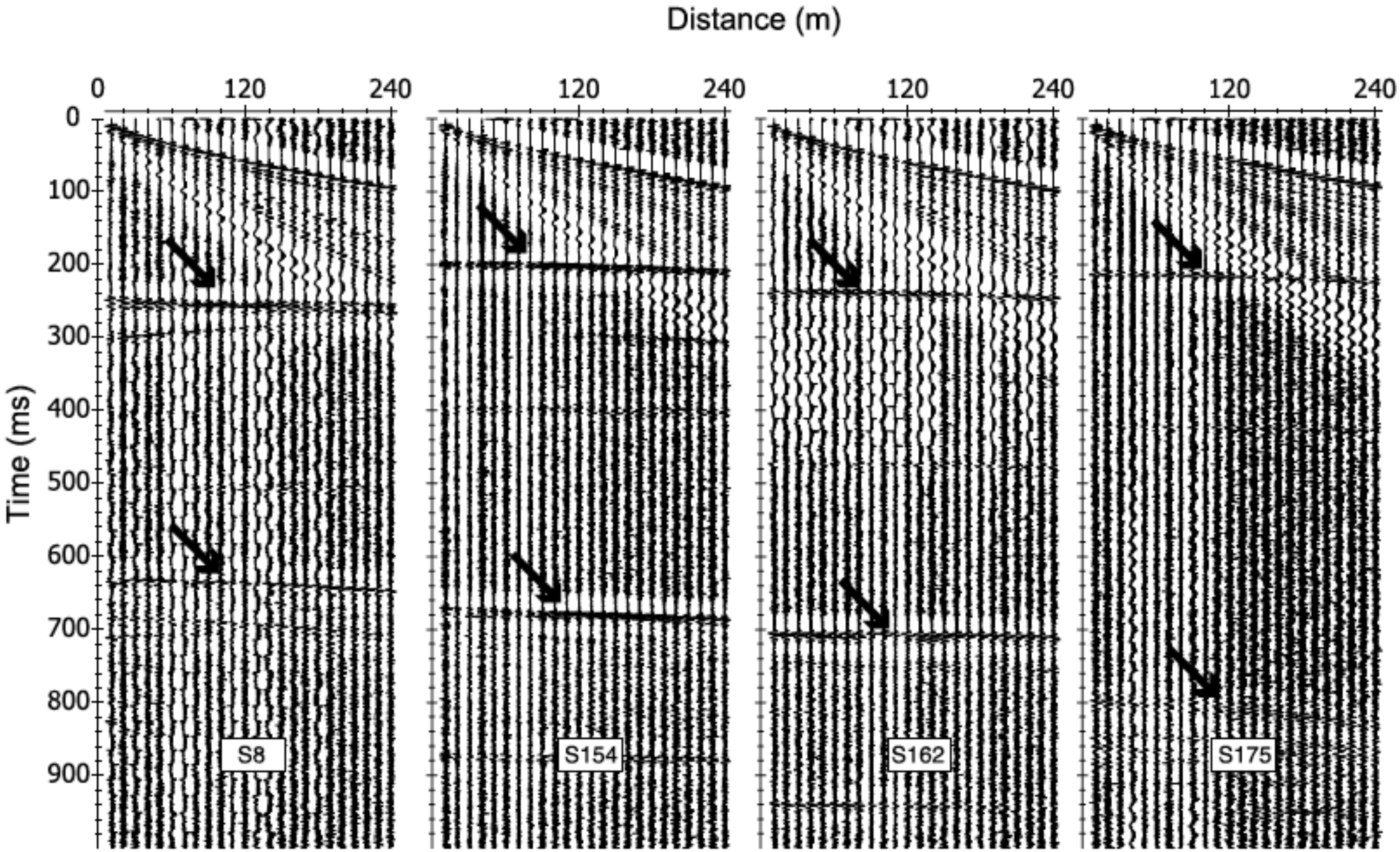


D. Steinhage, AWI

Observational Methods

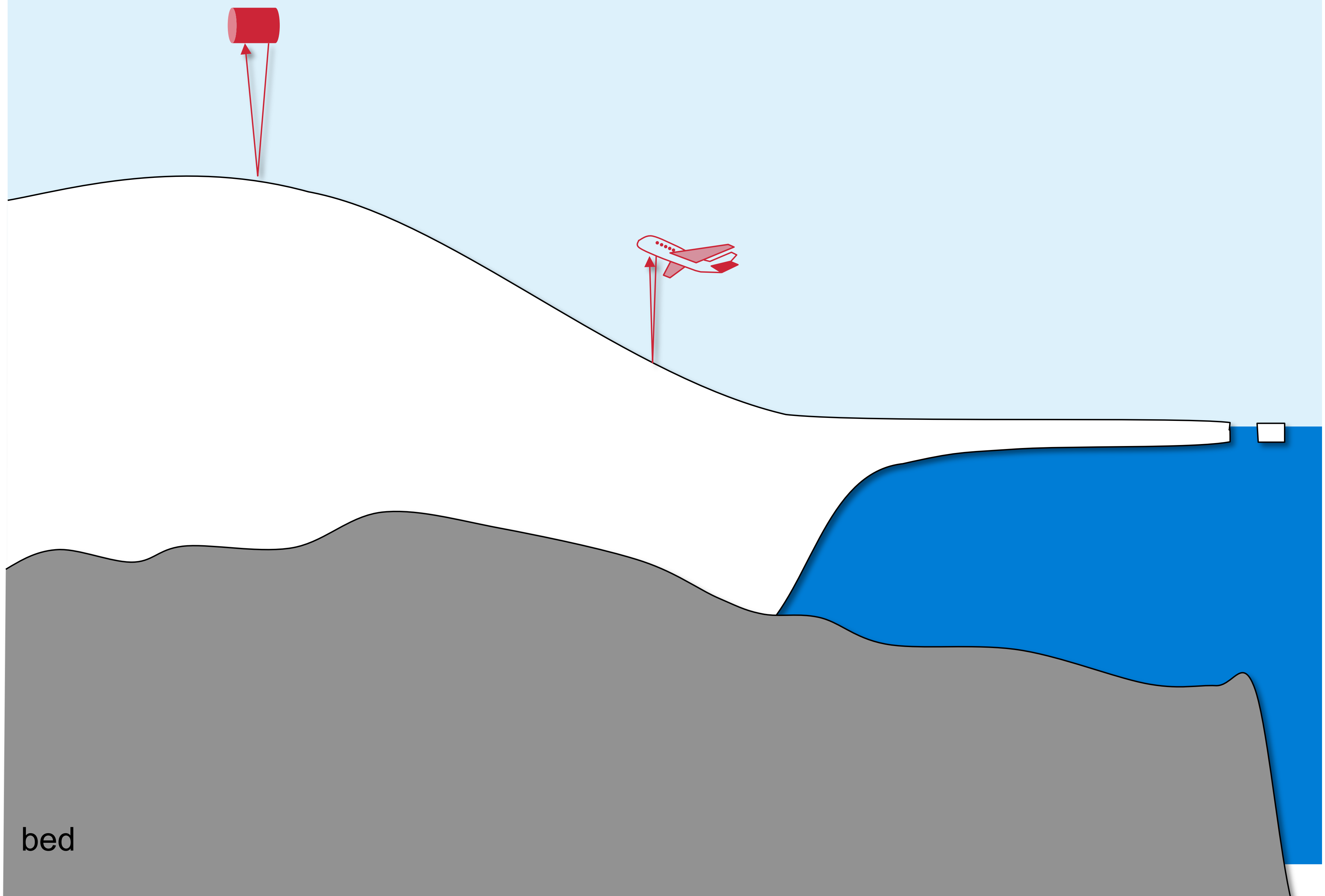


Seismics

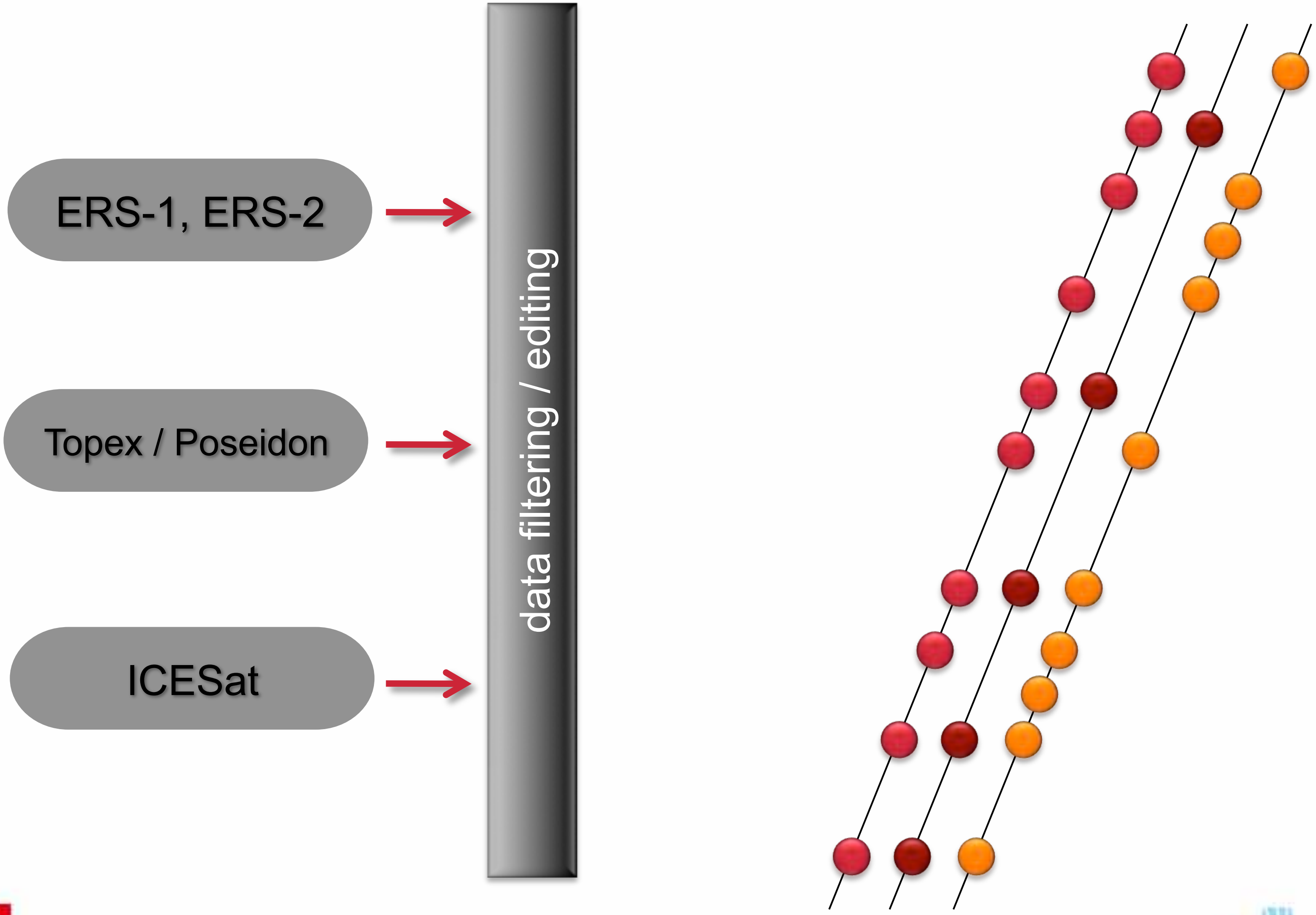


Nøst, 2004

Observational Methods



Altimeters – satellite based



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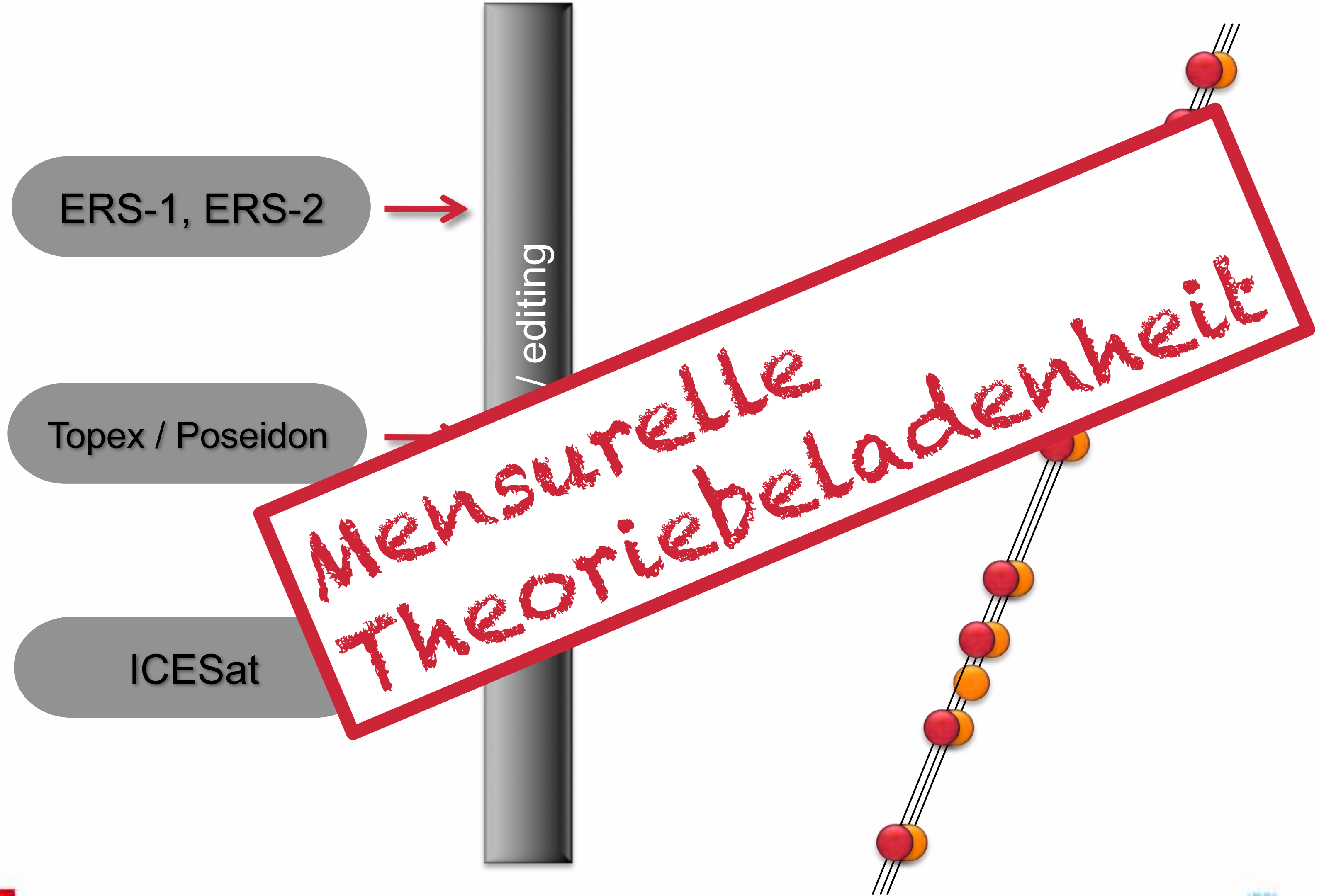
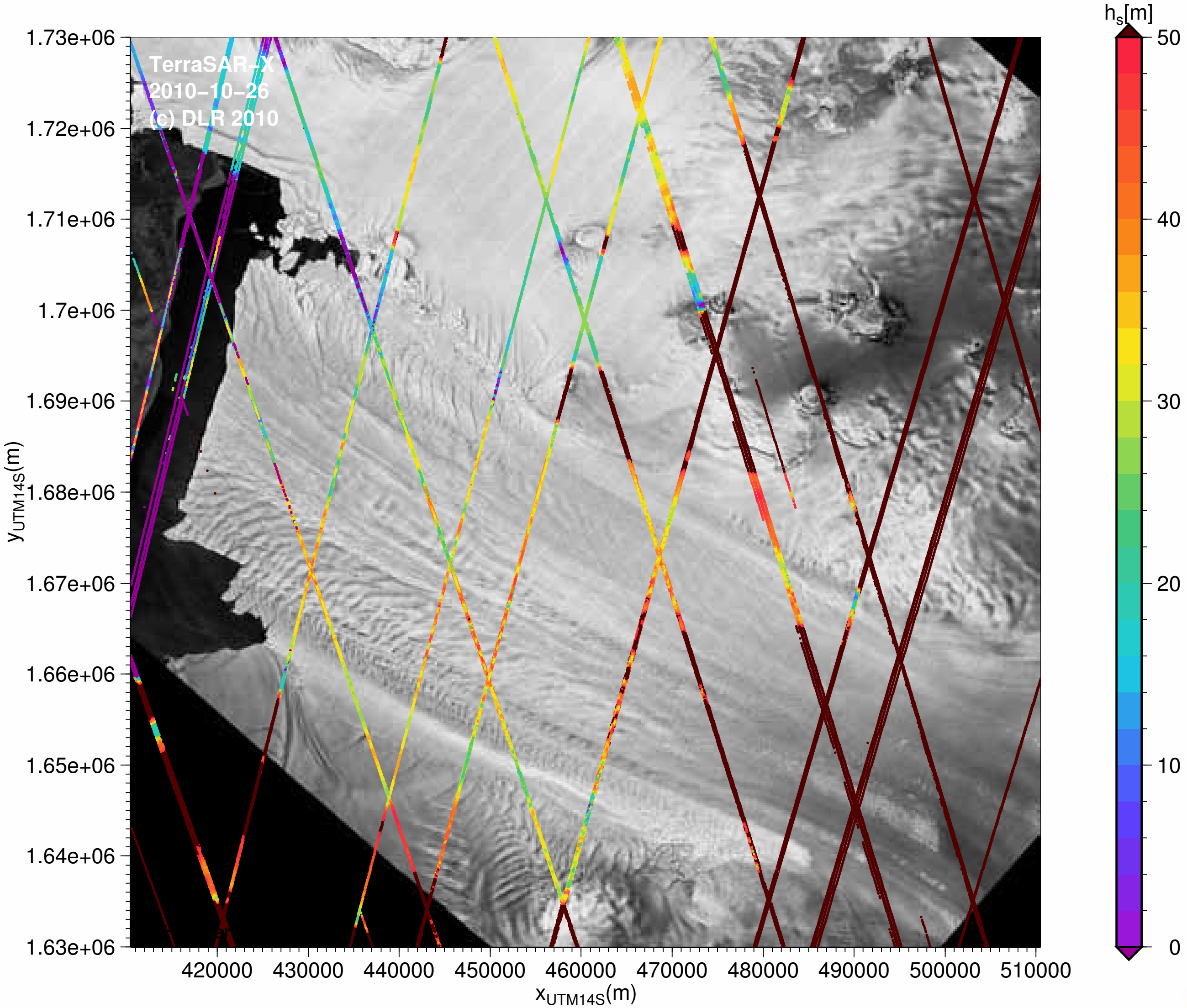
Altimeters – satellite based



Surface elevation



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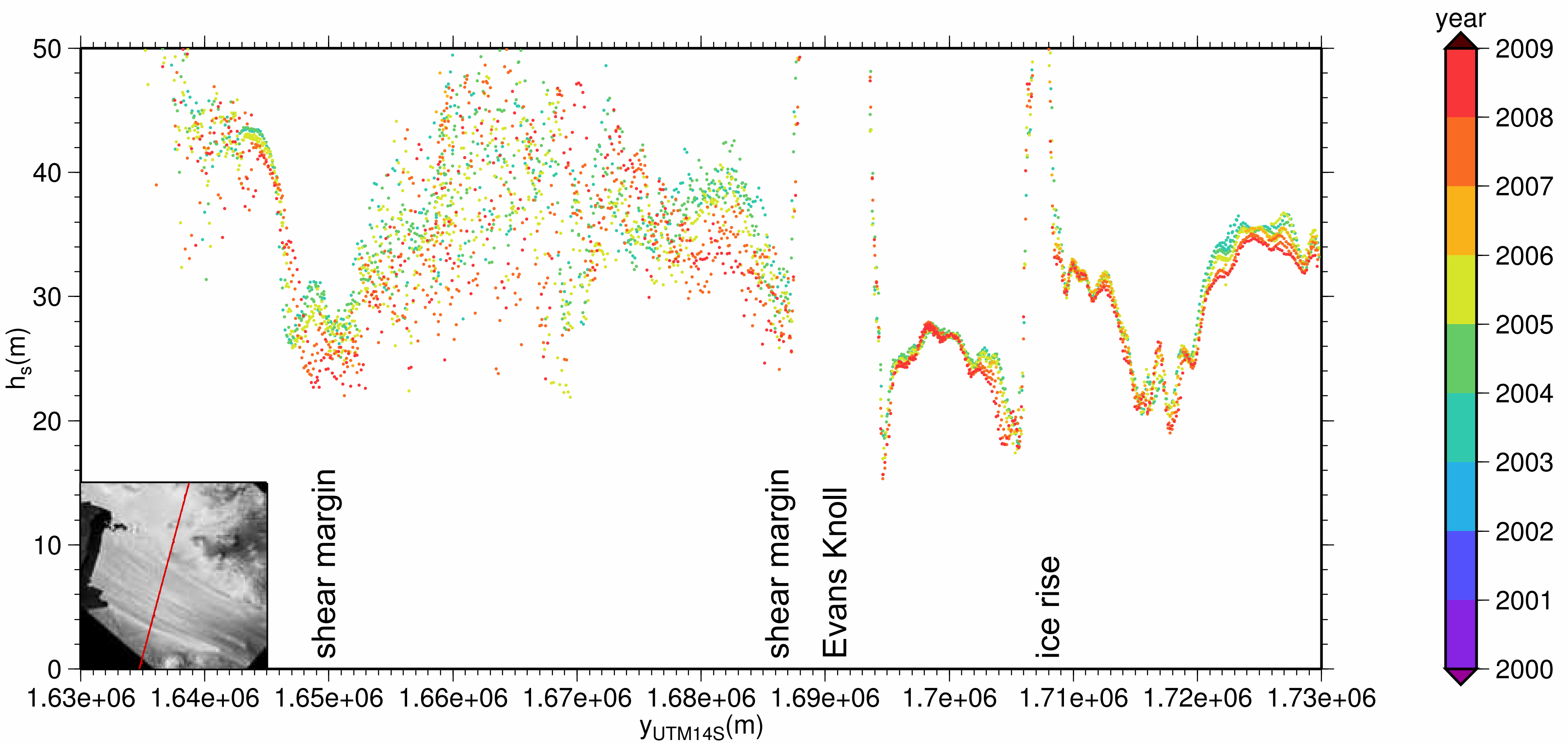
Changes in the last decade



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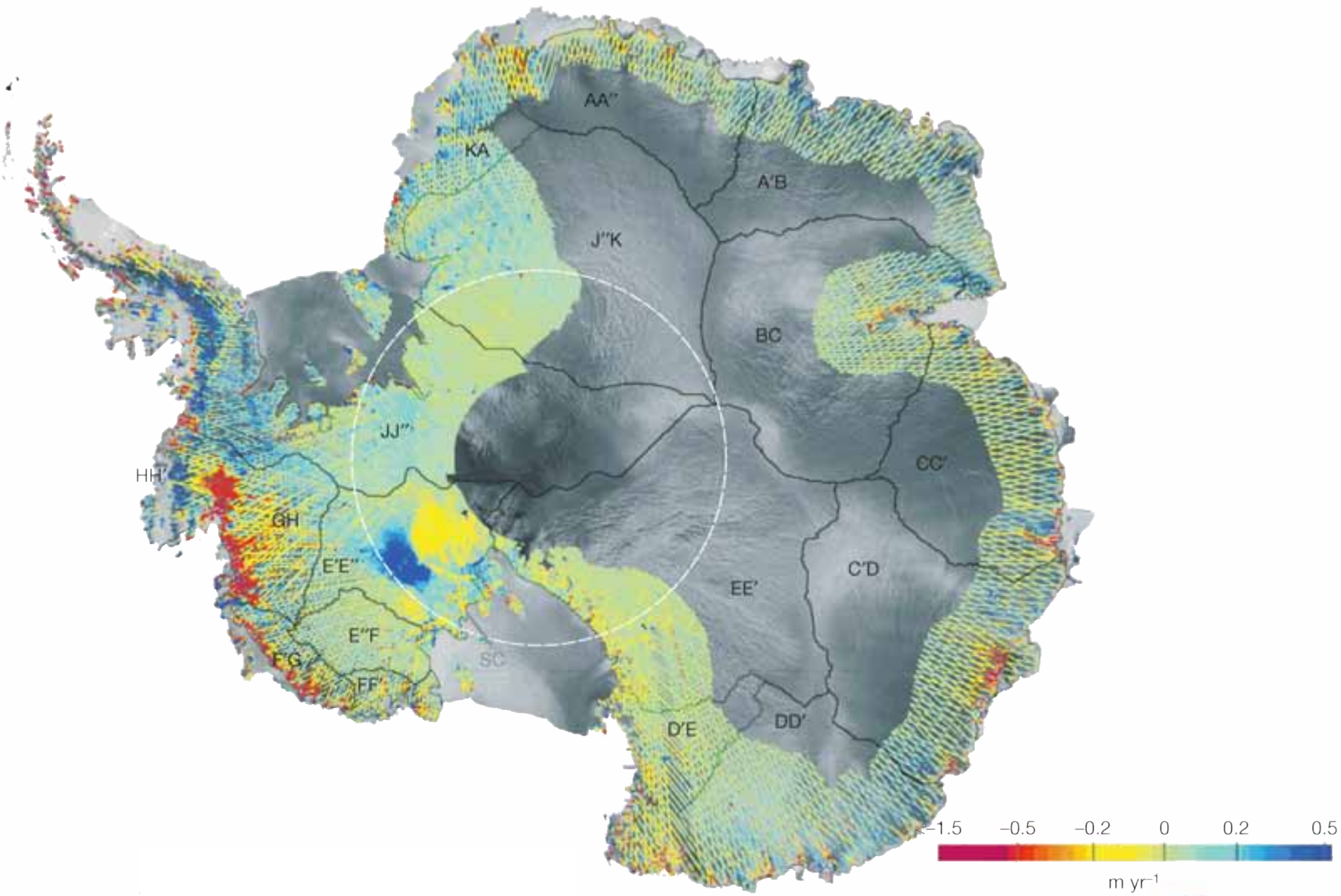
Data coverage – satellite altimetry



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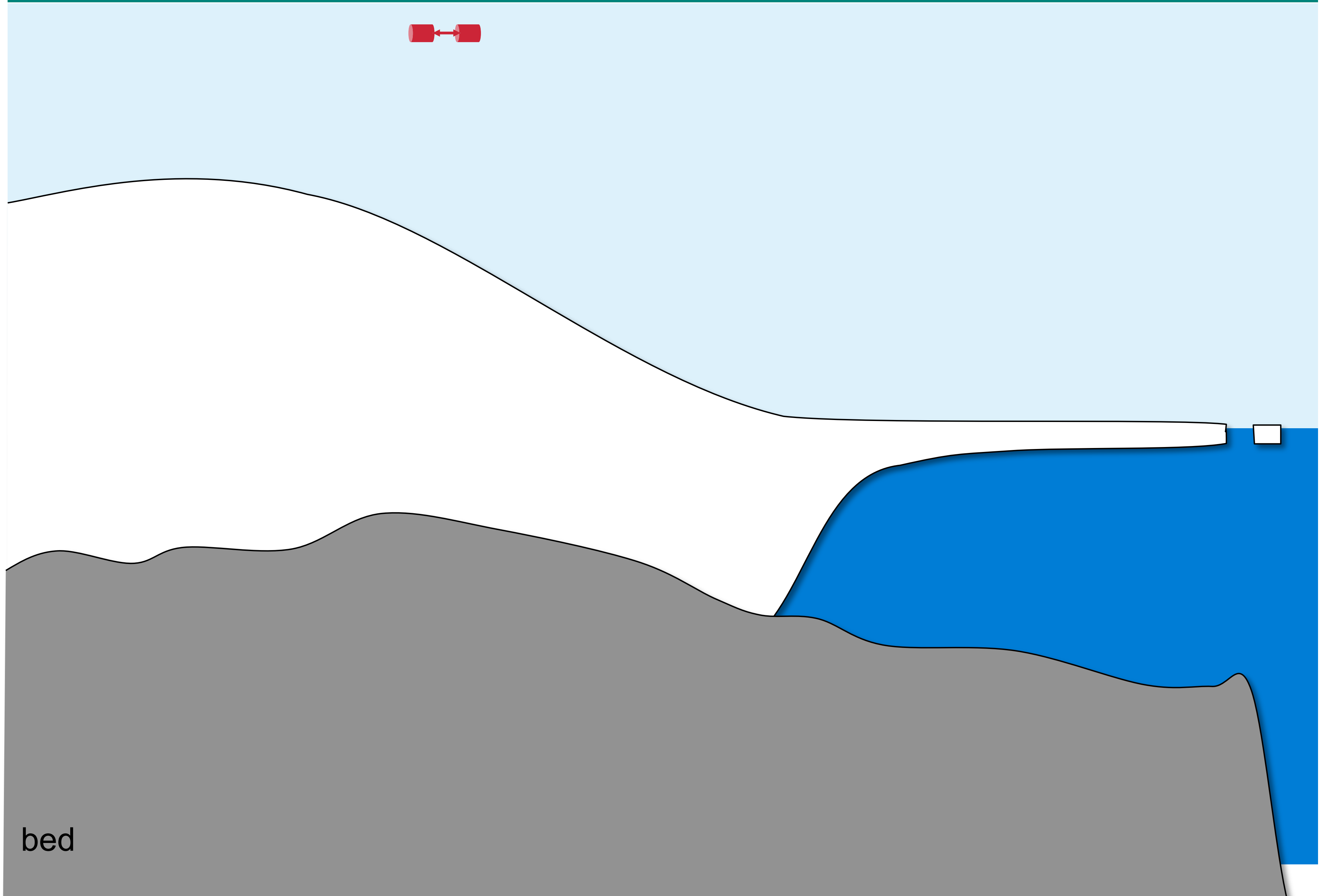
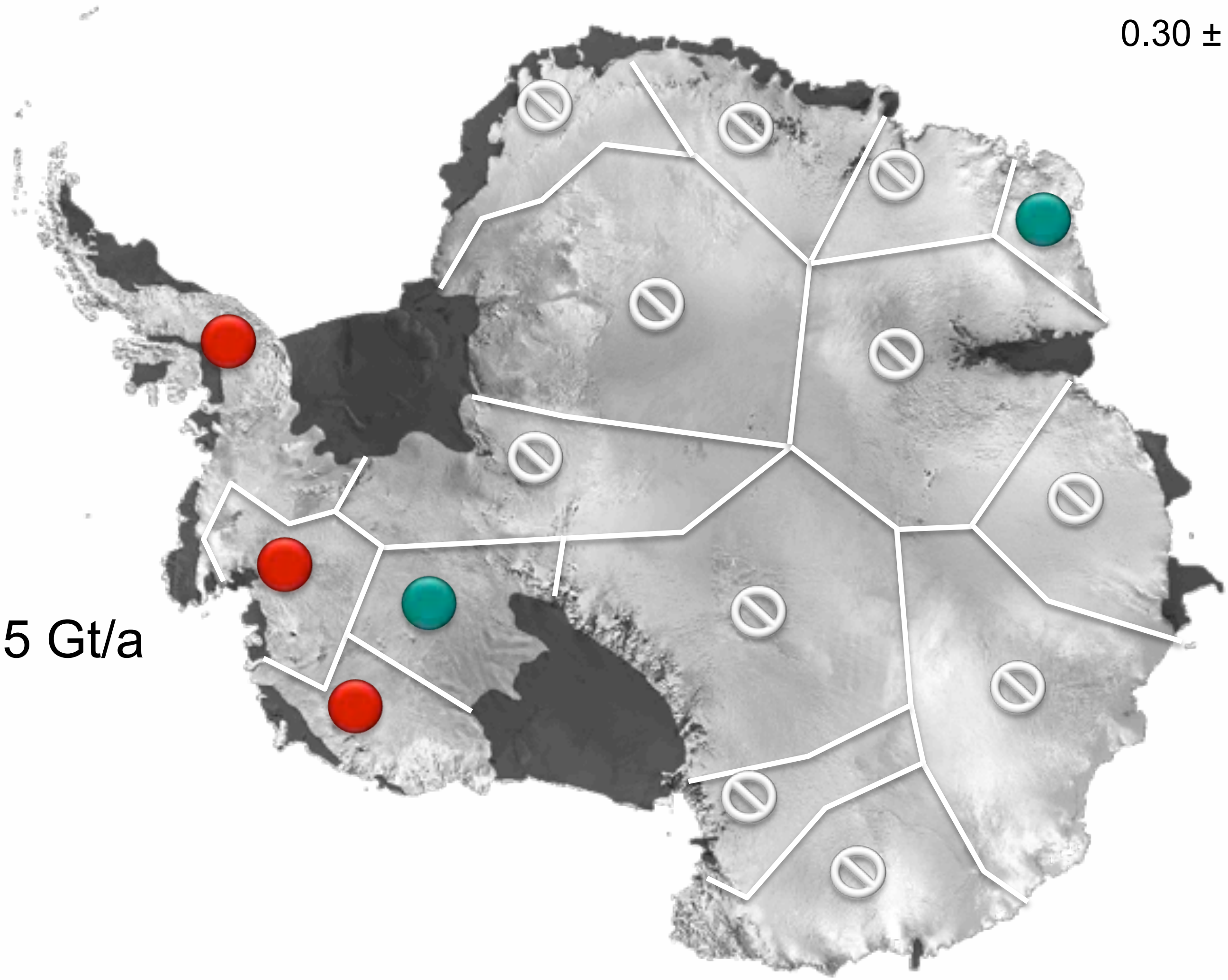
Pritchard et al., 2009

Observational Methods



Data coverage – mass change



-109 ± 48 Gt/a

0.30 ± 0.13 mm/a SLE

-105 ±35 Gt/a

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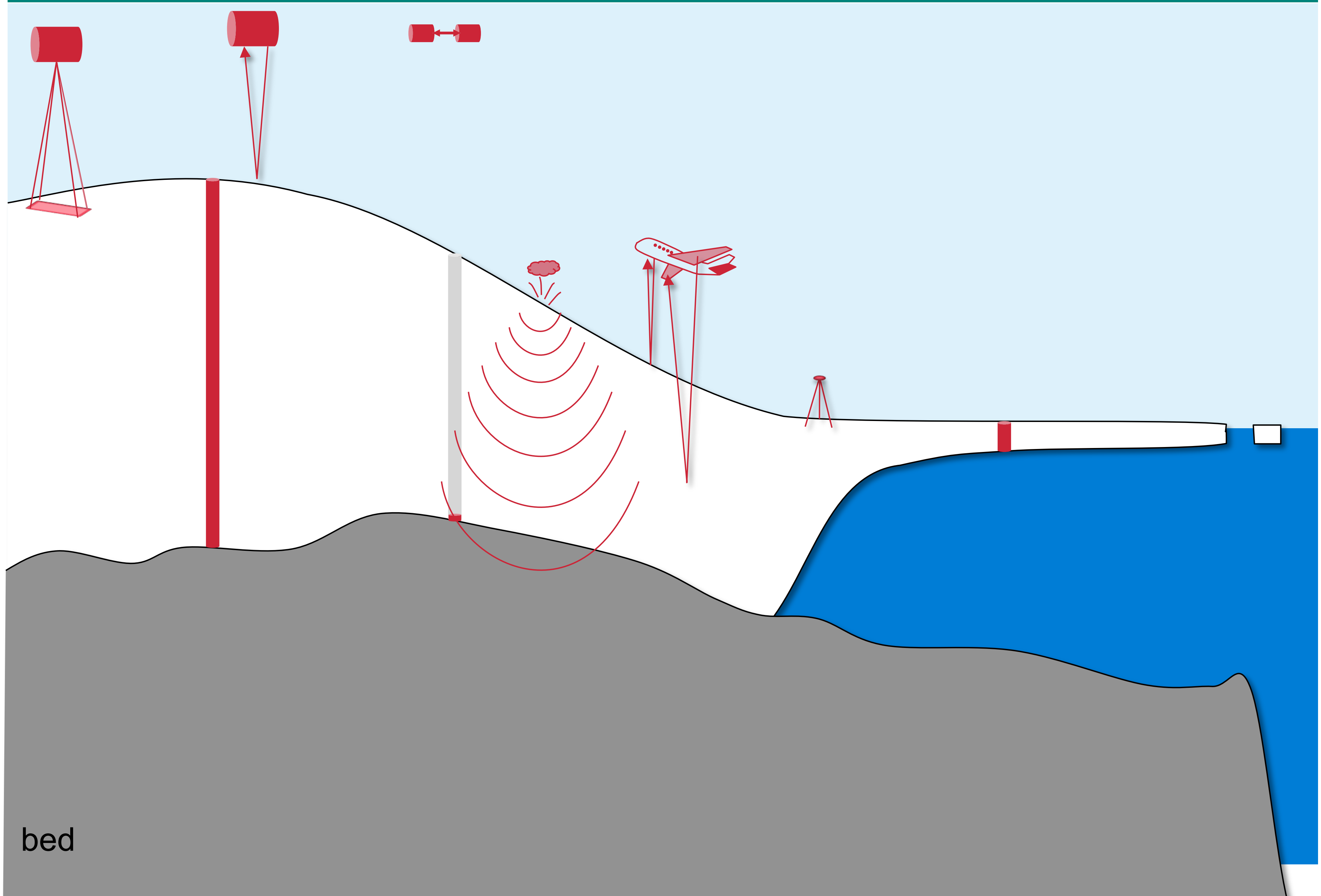
Horwath & Dietrich, 2009

Observational Methods



GPS observations of horizontal and vertical position

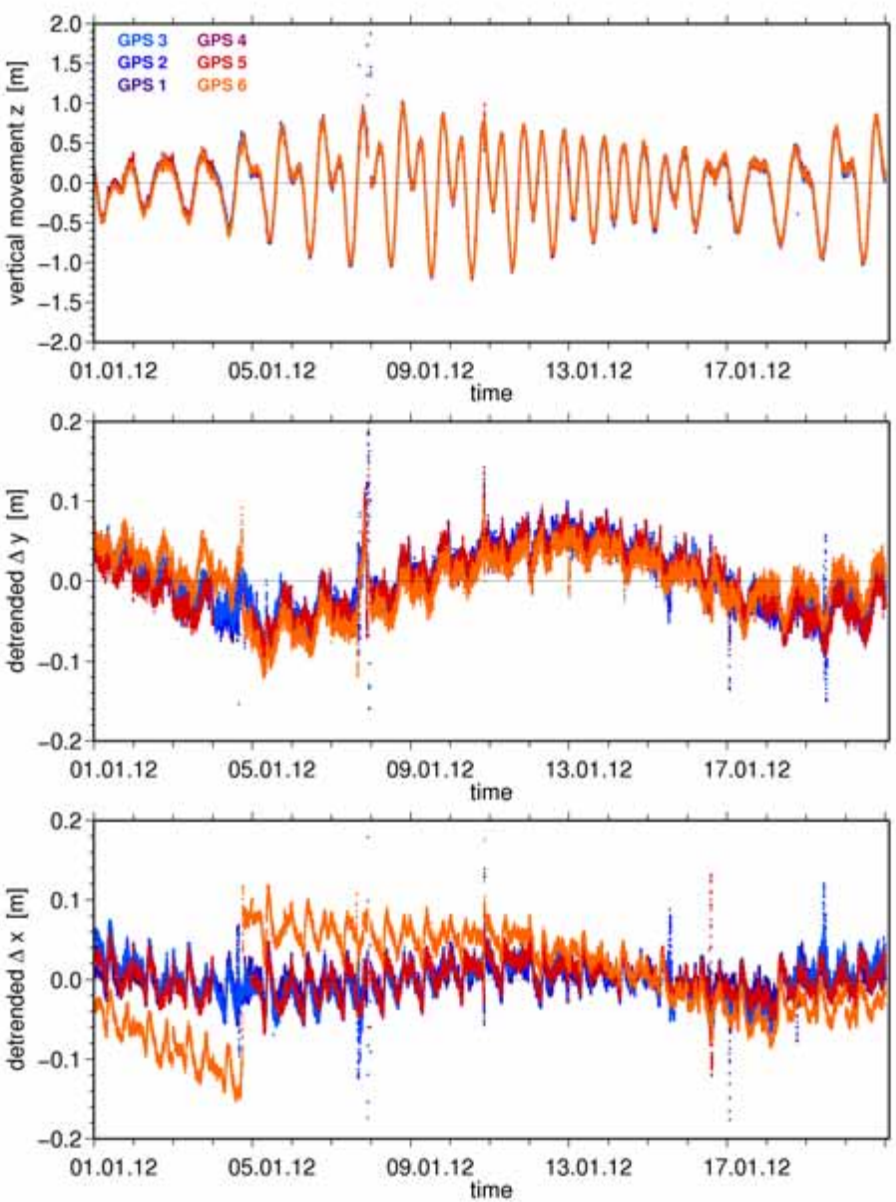
© J. Lohse

GPS observations of horizontal and vertical position

vertical: z

detrended horizontal: y

detrended horizontal: x



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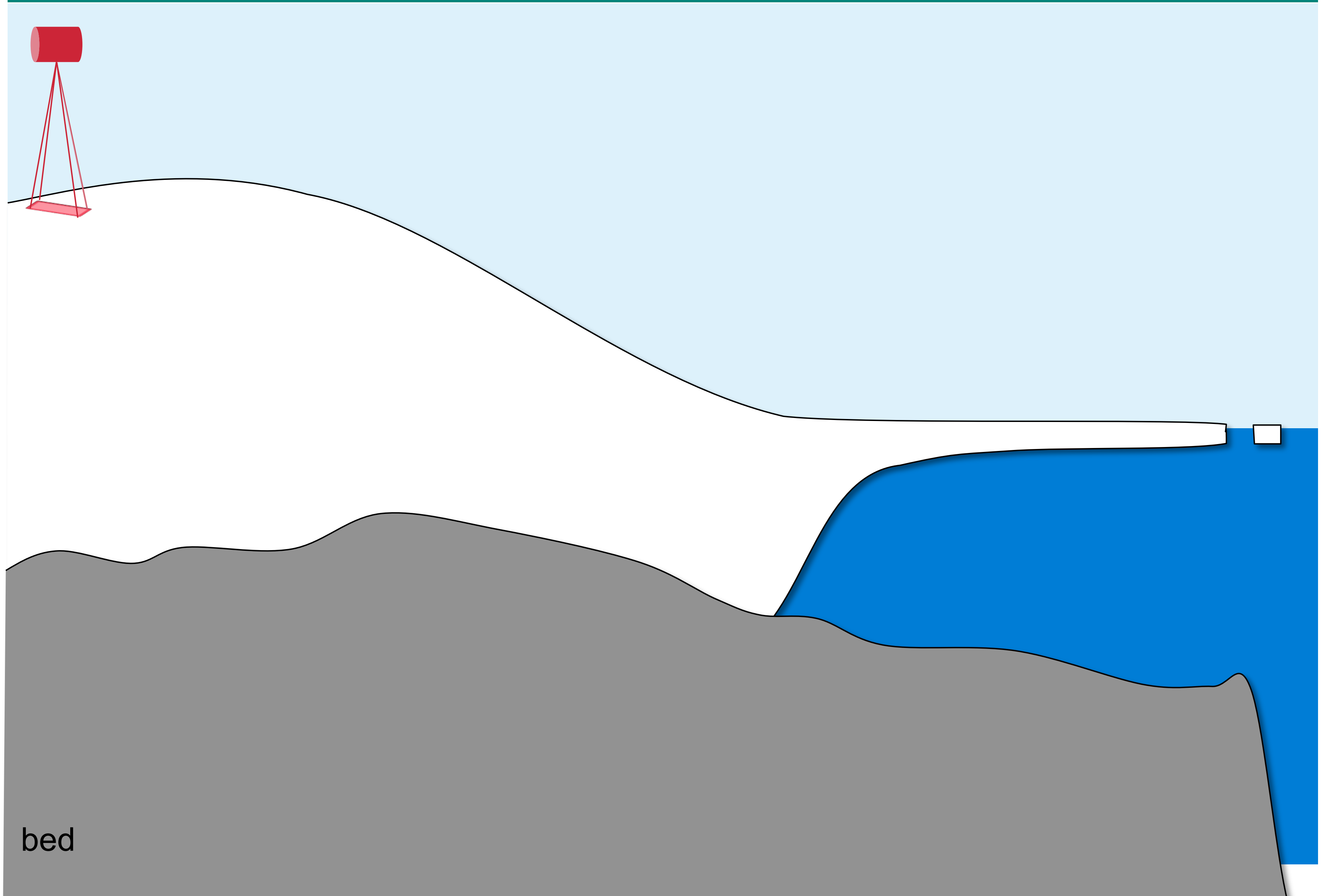
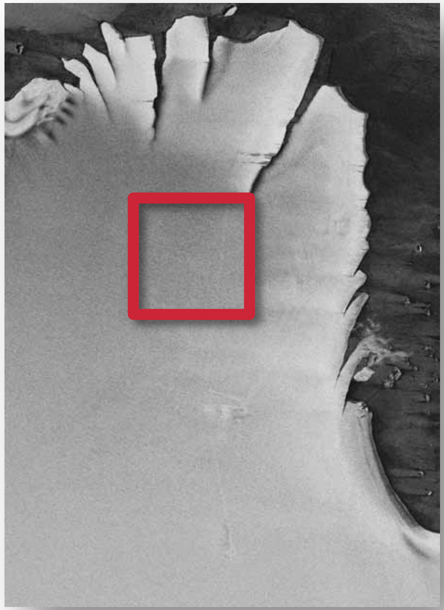
Observational Methods



Flow velocities from remote sensing – feature tracking



t_0
reference chip

t_1
search area chip

calculate the correlation-index between the reference chip and the search-area chip → select the chip with the largest correlation

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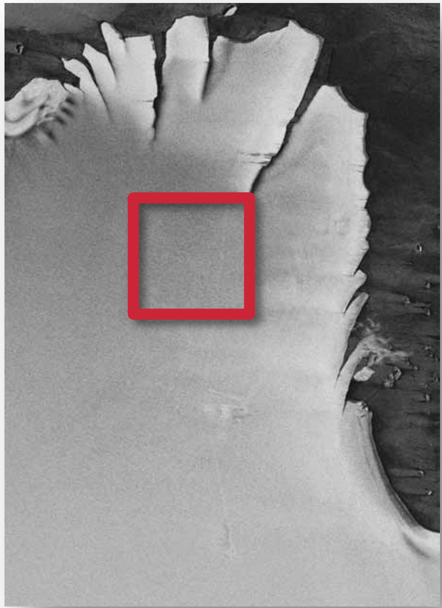
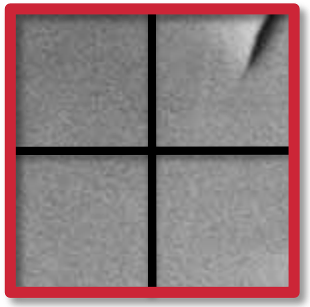
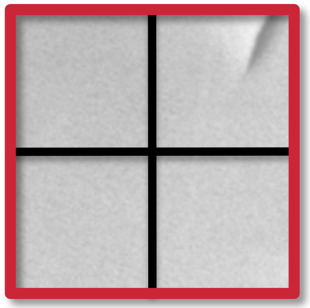
Flow velocities from remote sensing – speckle tracking



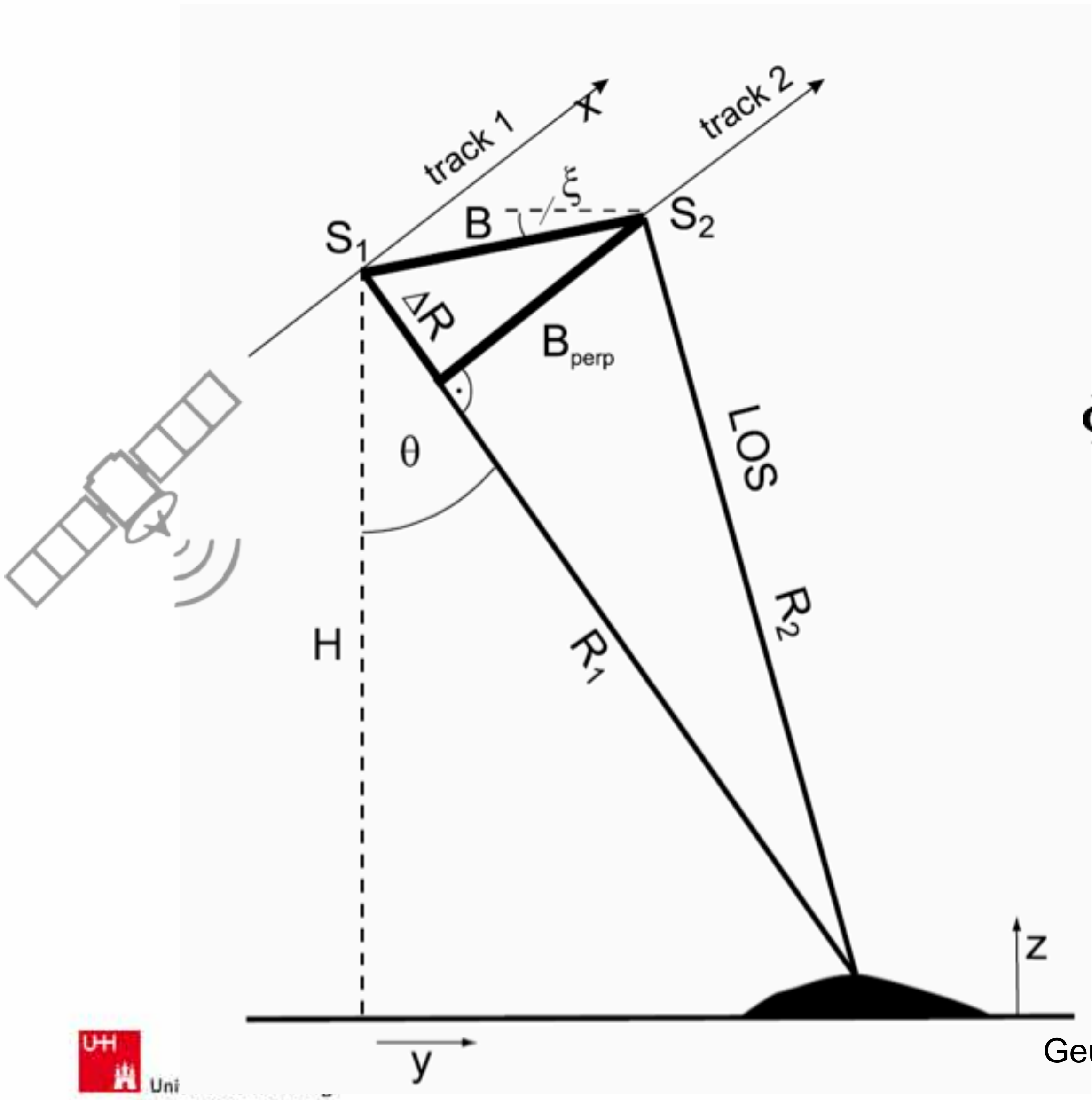
t_0
master

t_1
slave





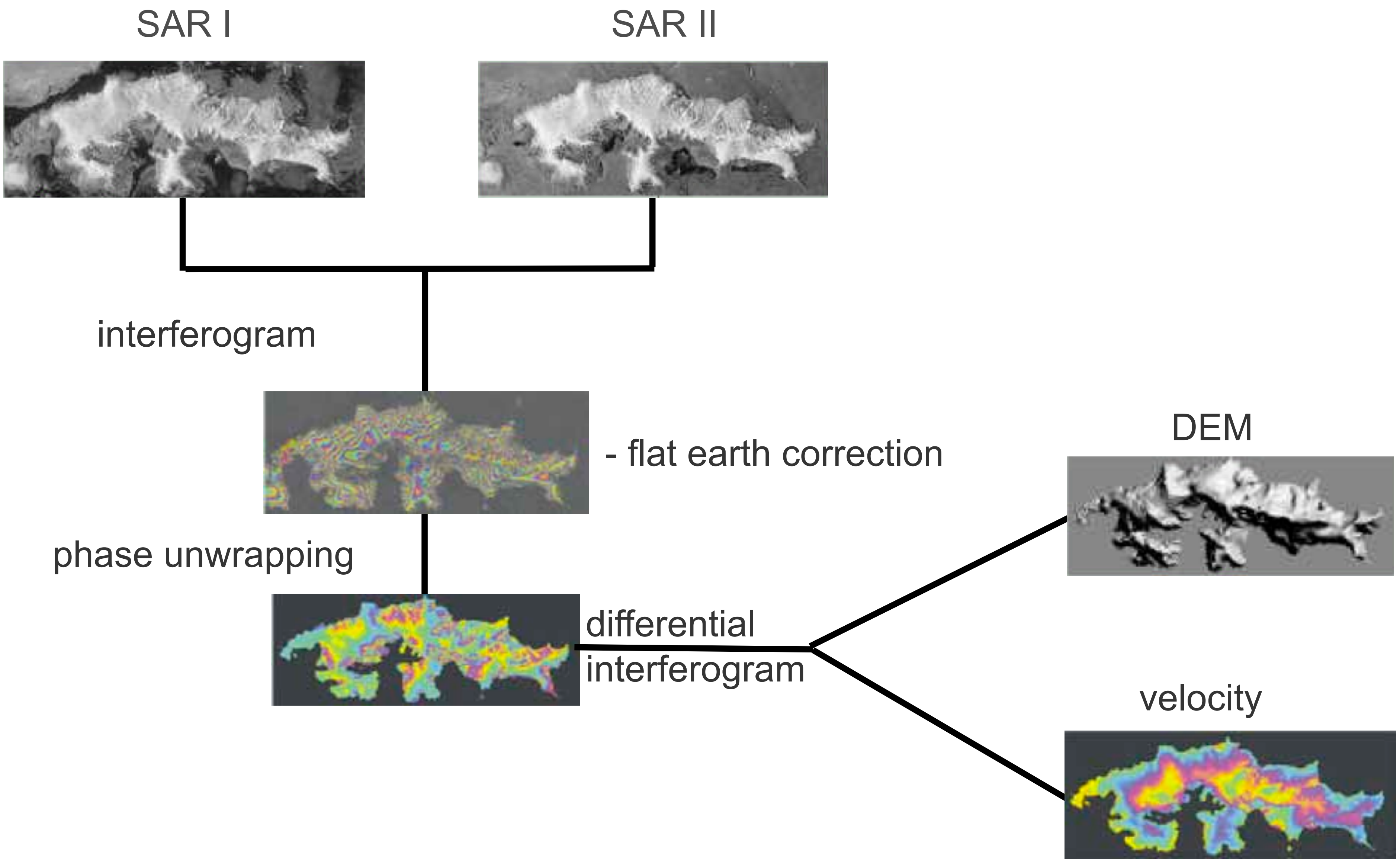
Flow velocities from remote sensing – interferometry



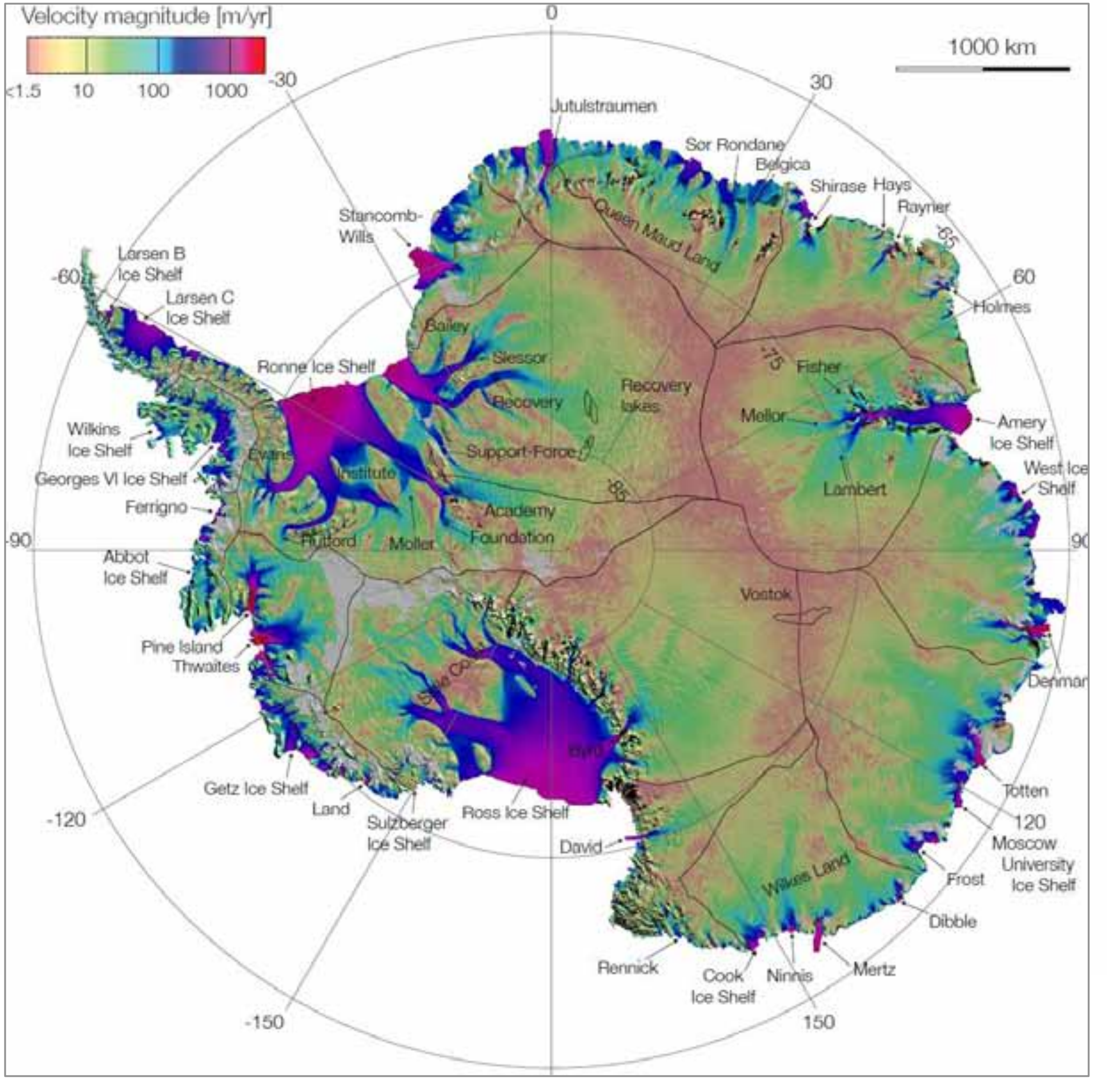
$$\Phi = \frac{4\pi}{\lambda}\Delta R$$

Geudtner 1995

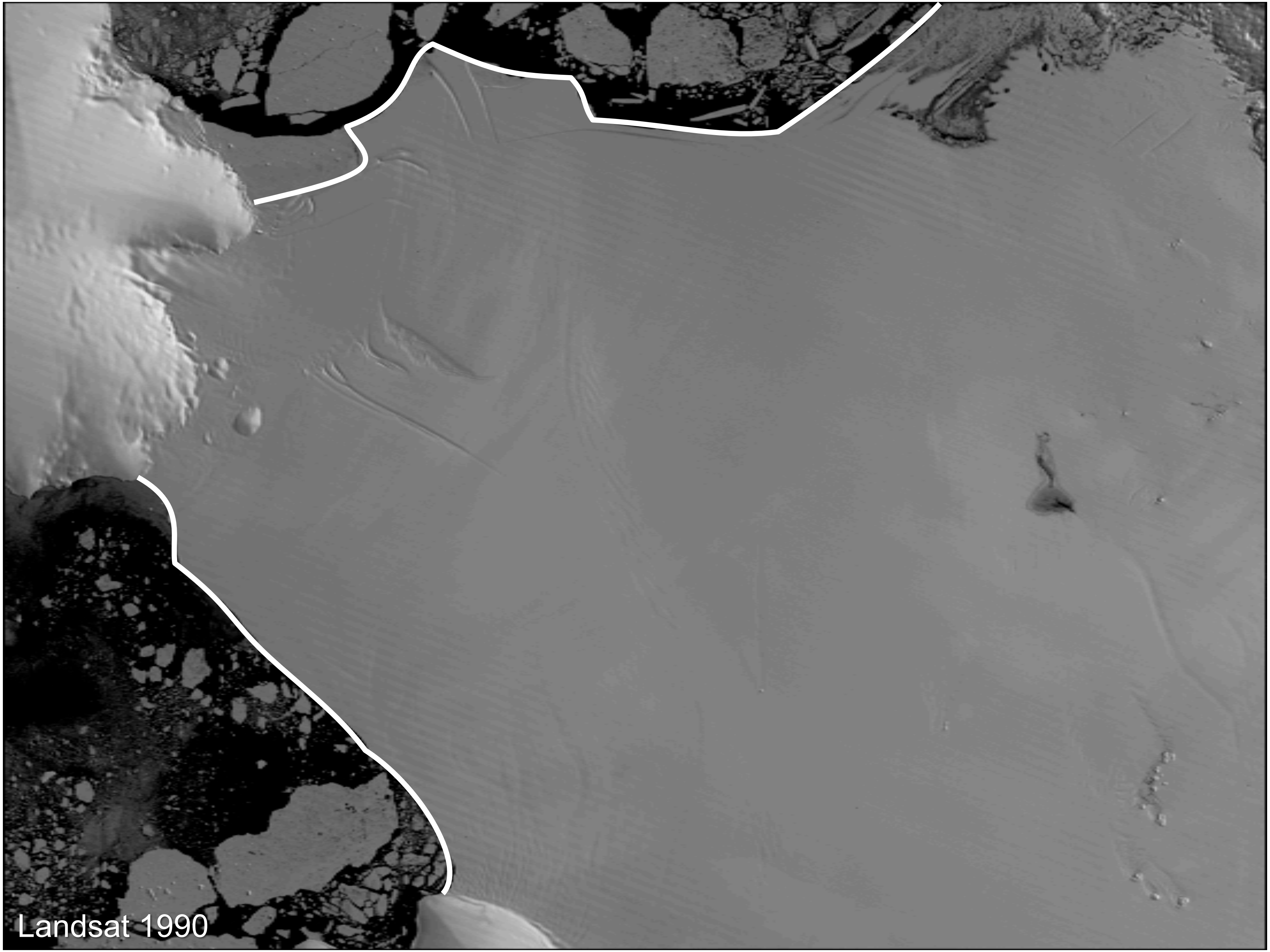
Flow velocities from remote sensing – interferometry

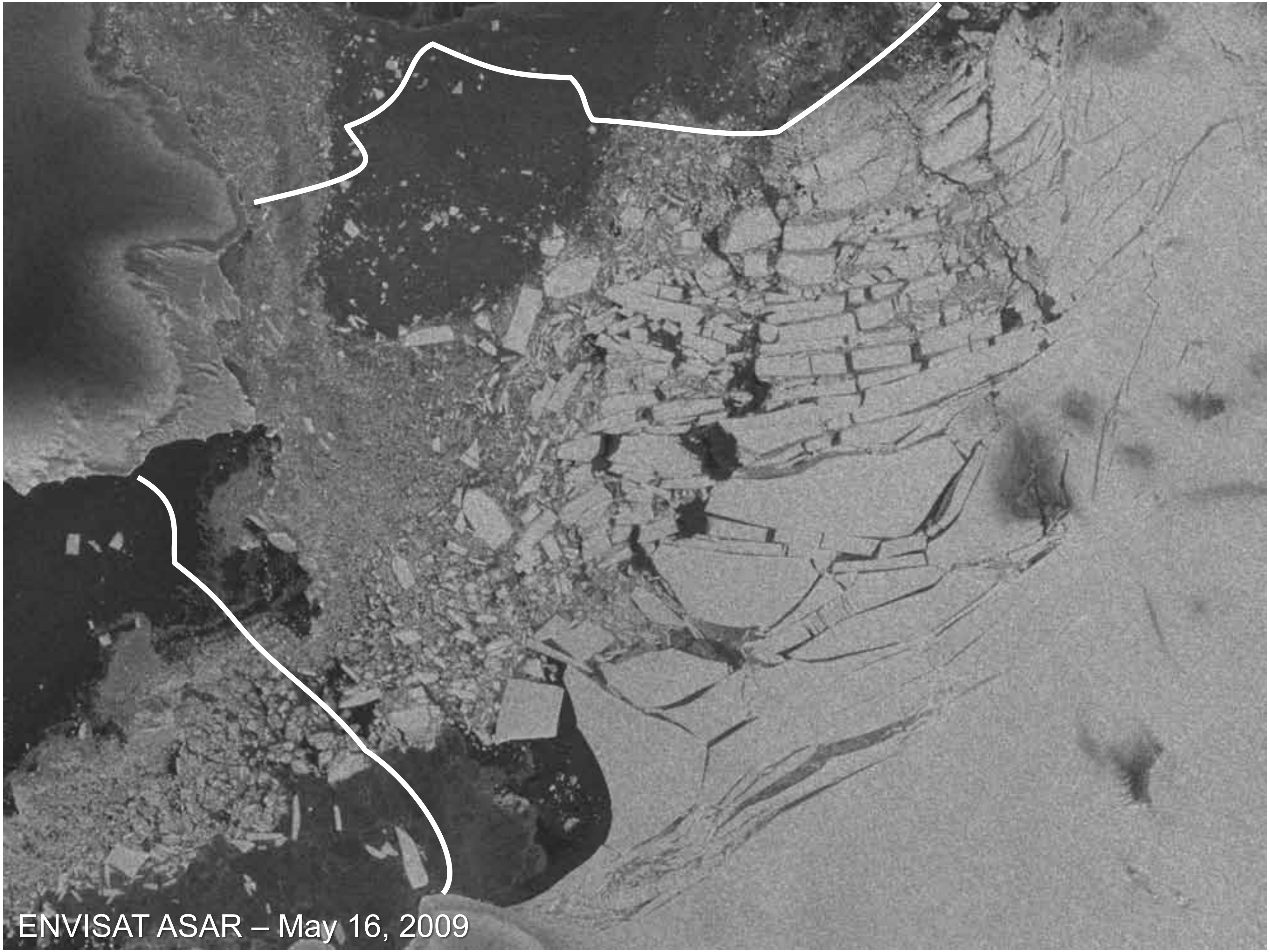


Data coverage – surface velocities



© NASA/JPL-Caltech/UCI


Landsat 1990


ENVISAT ASAR – May 16, 2009


Pine Island Glacier, Antarctica
02 Oct 2011
TerraSAR-X
(c) DLR 2011
5 km

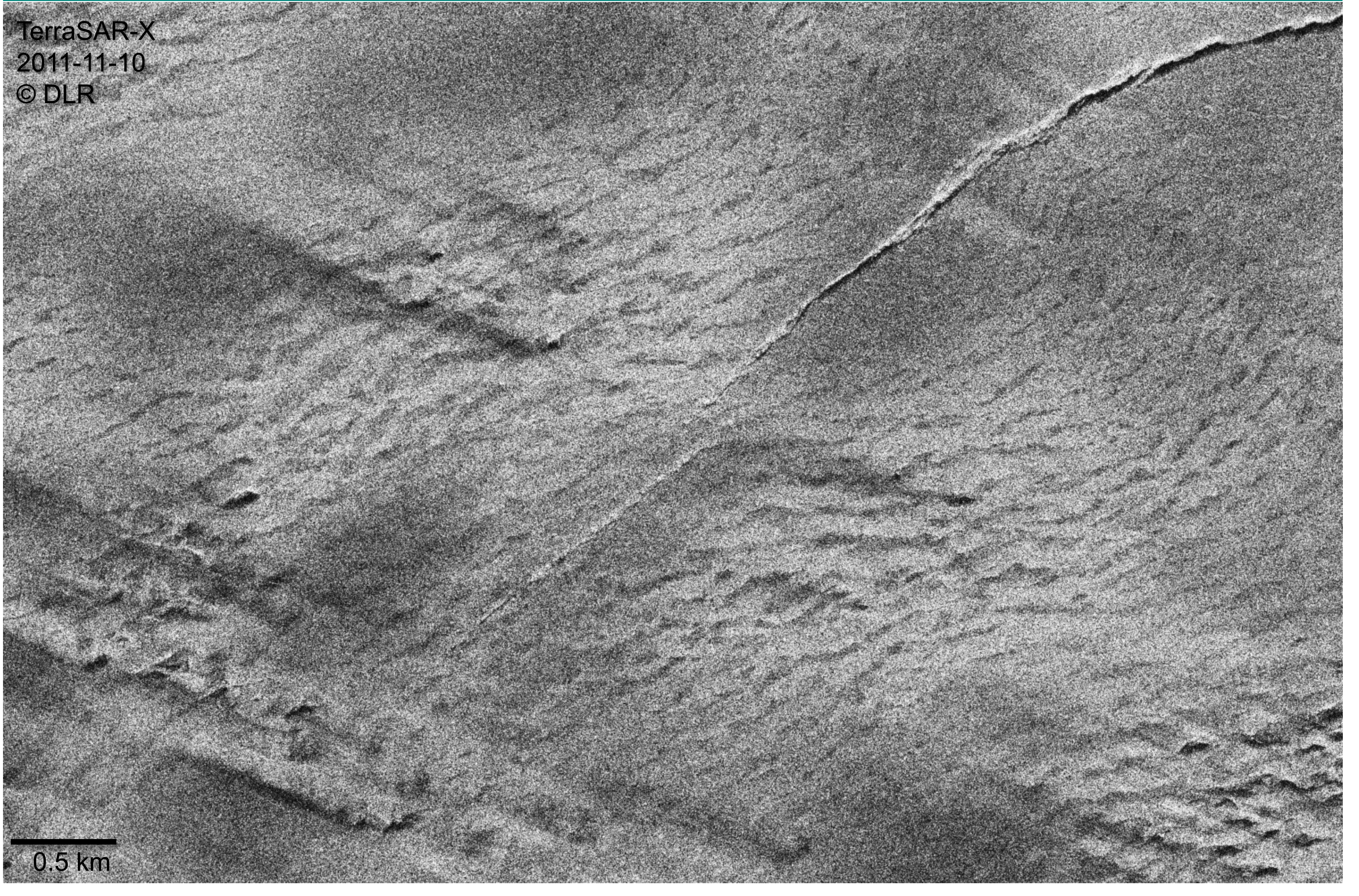

Pine Island Glacier, Antarctica
20 Jan 2012
TerraSAR-X
(c) DLR 2011
5 km

Pine Island Rift

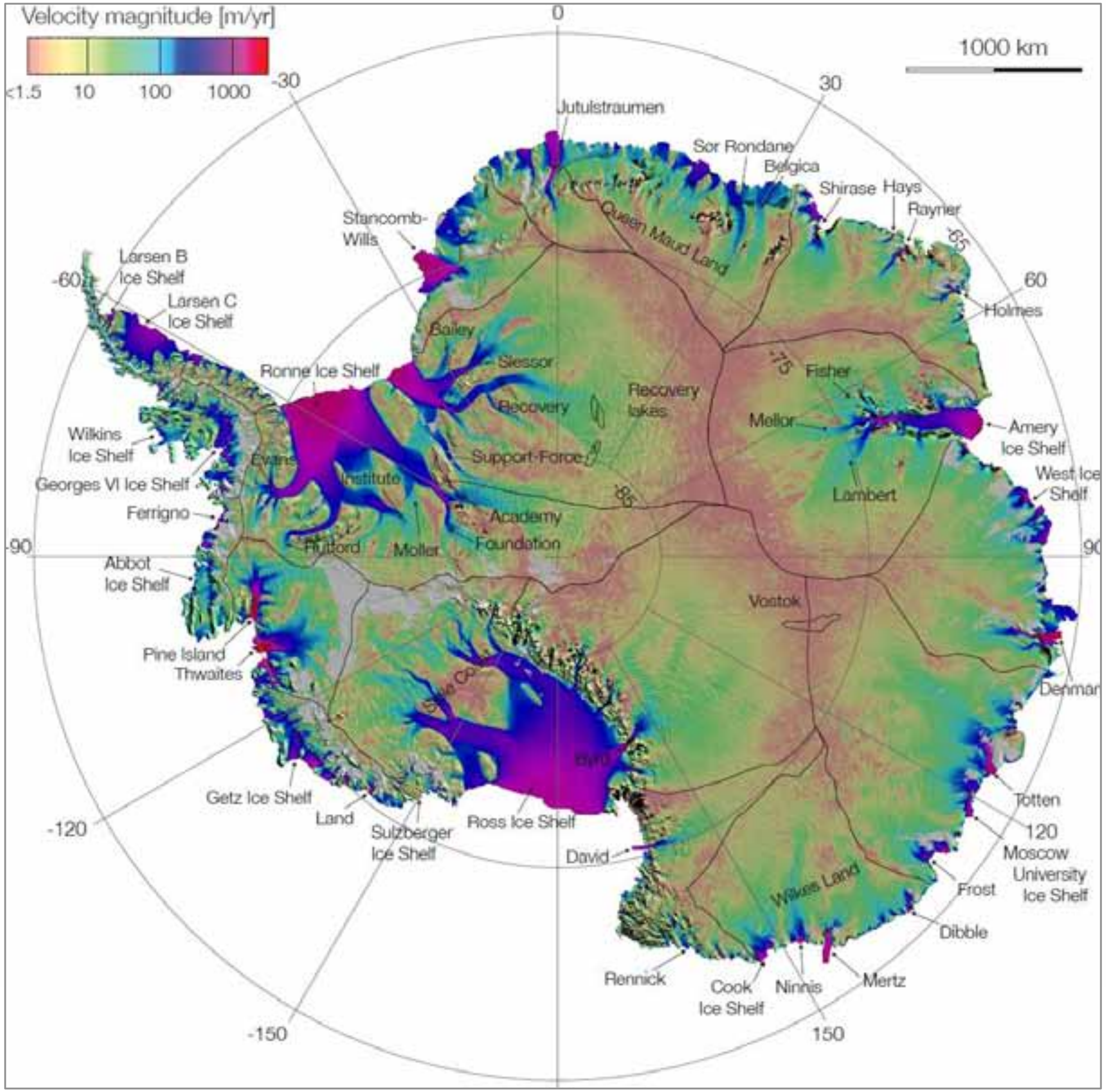
Pine Island Rift

Pine Island Rift

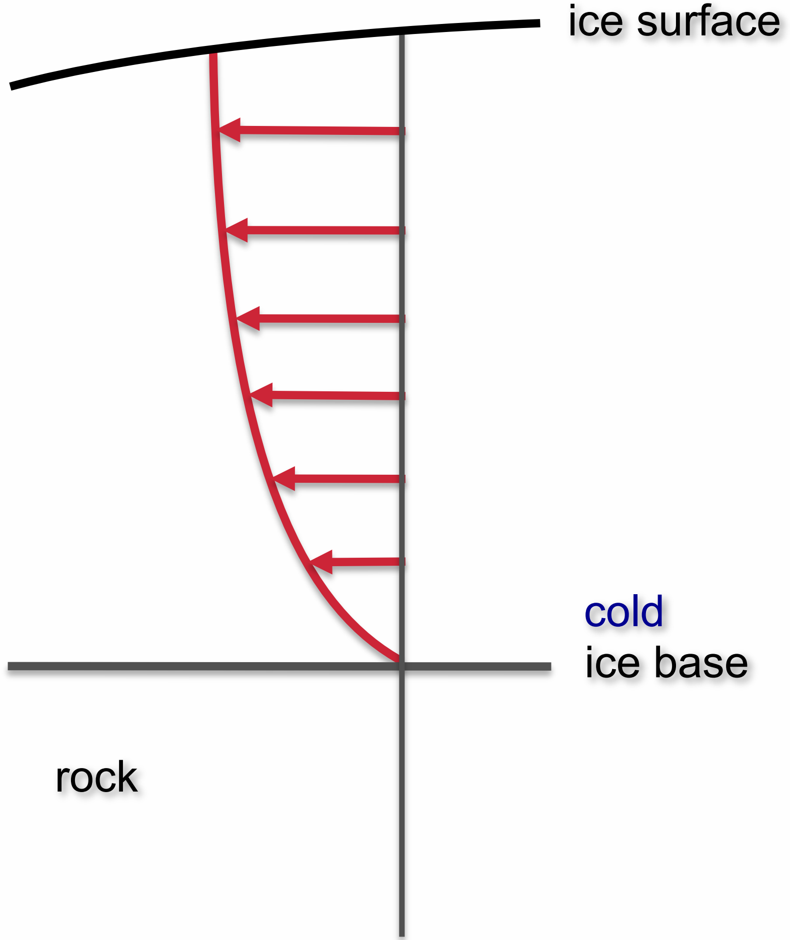
Pine Island Rift



Data coverage – surface velocities



Horizontal velocities



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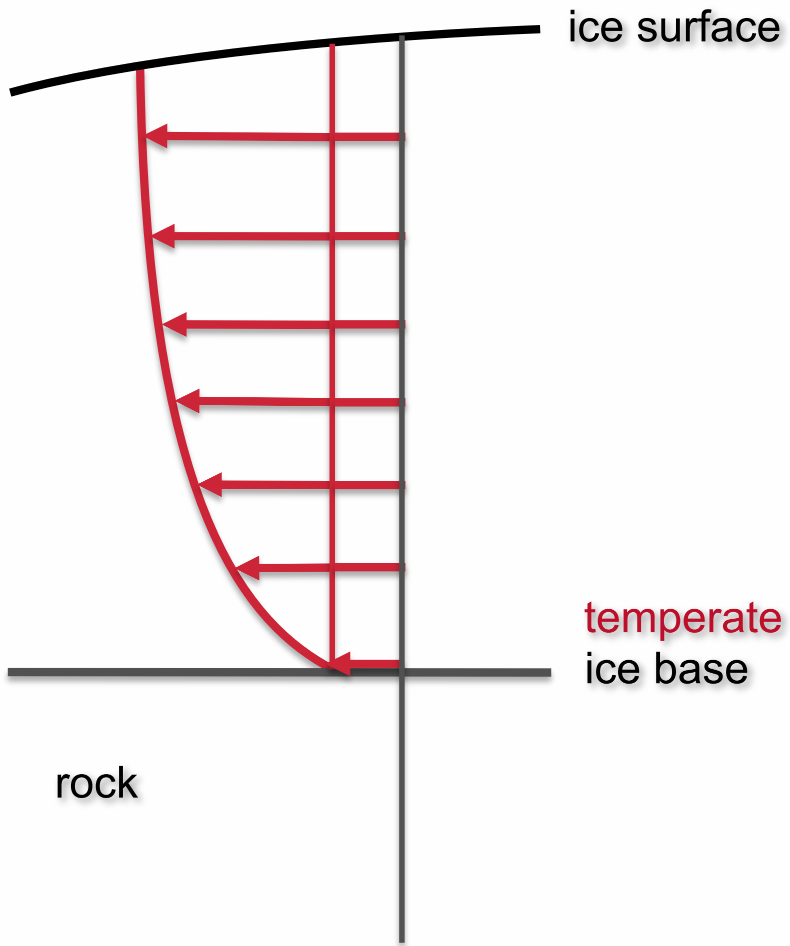
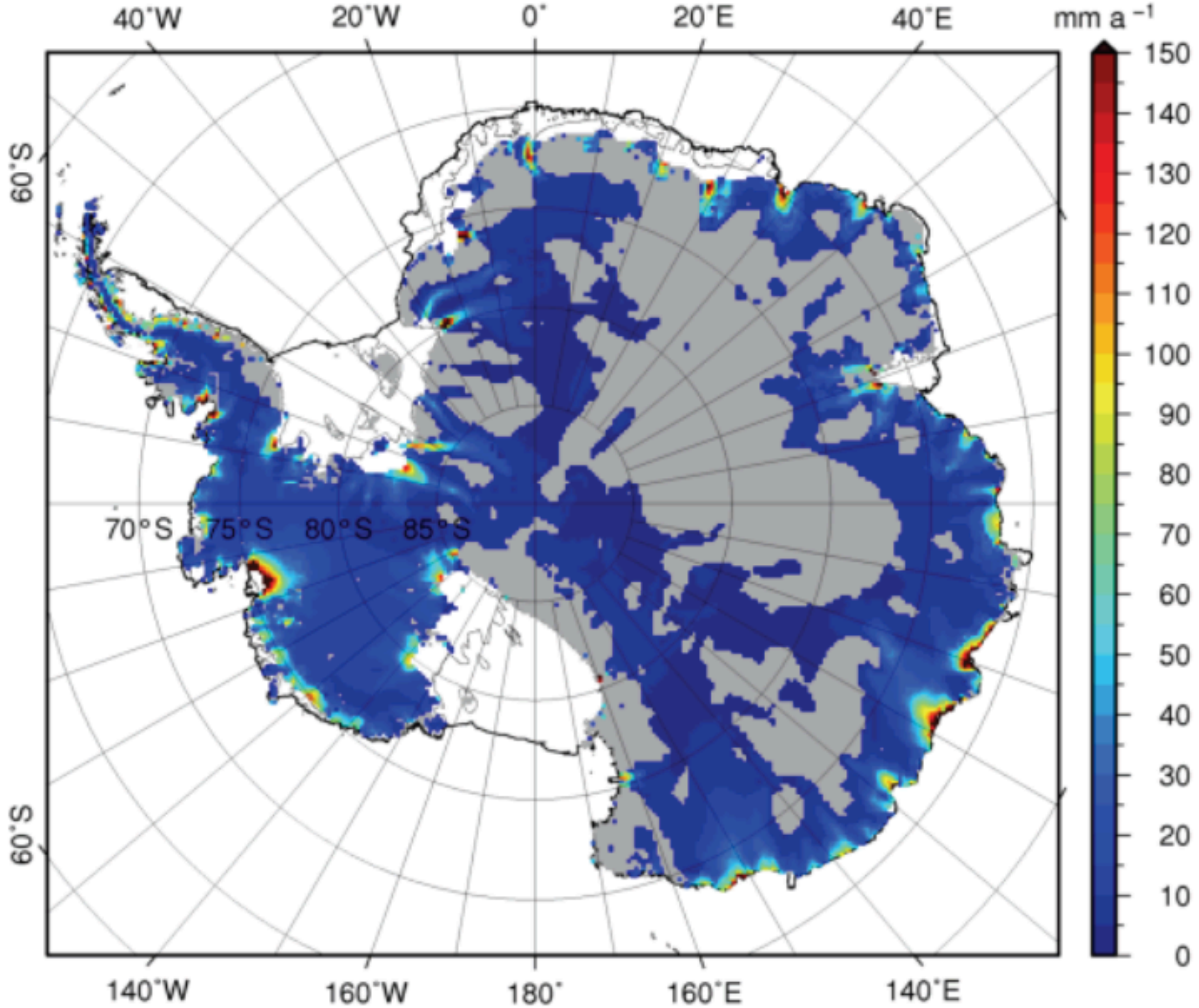
Horizontal velocities



Wet base



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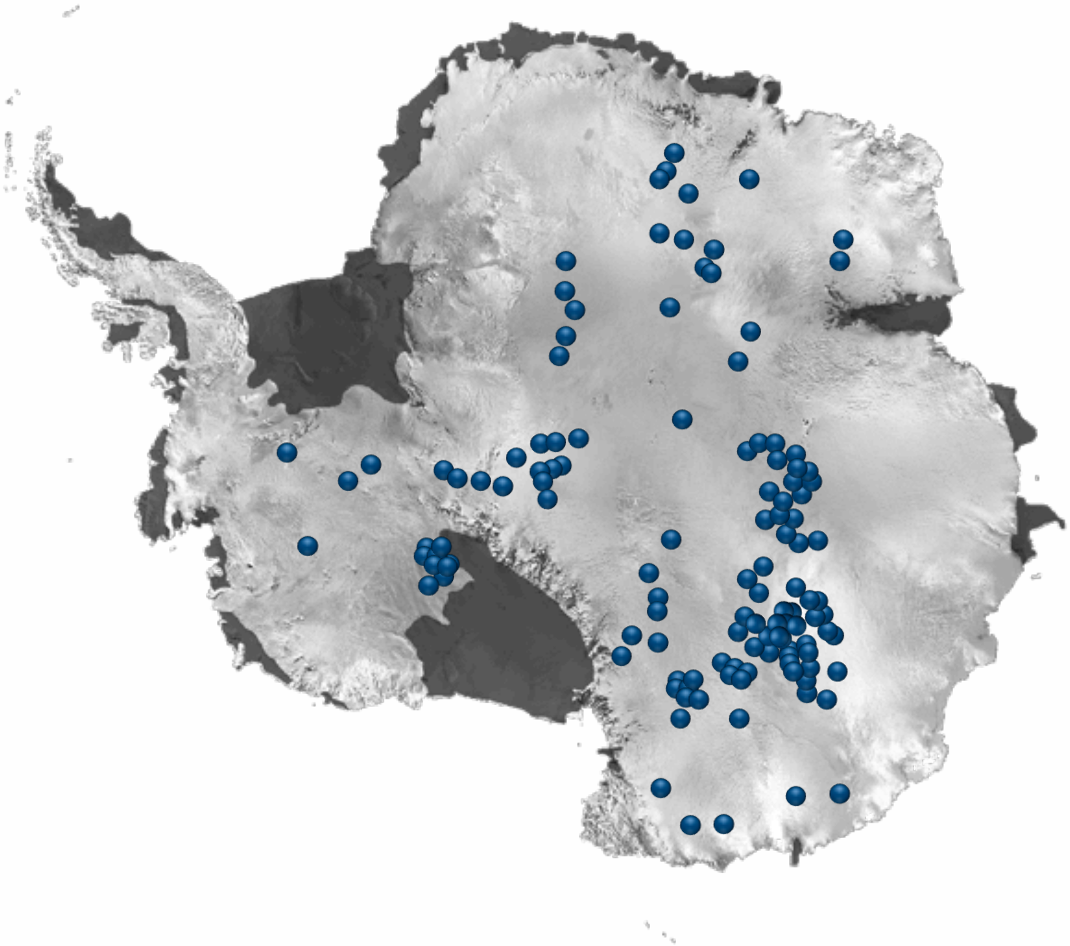
T. Kleiner

Subglacial lakes



after Siegert et al., 2005

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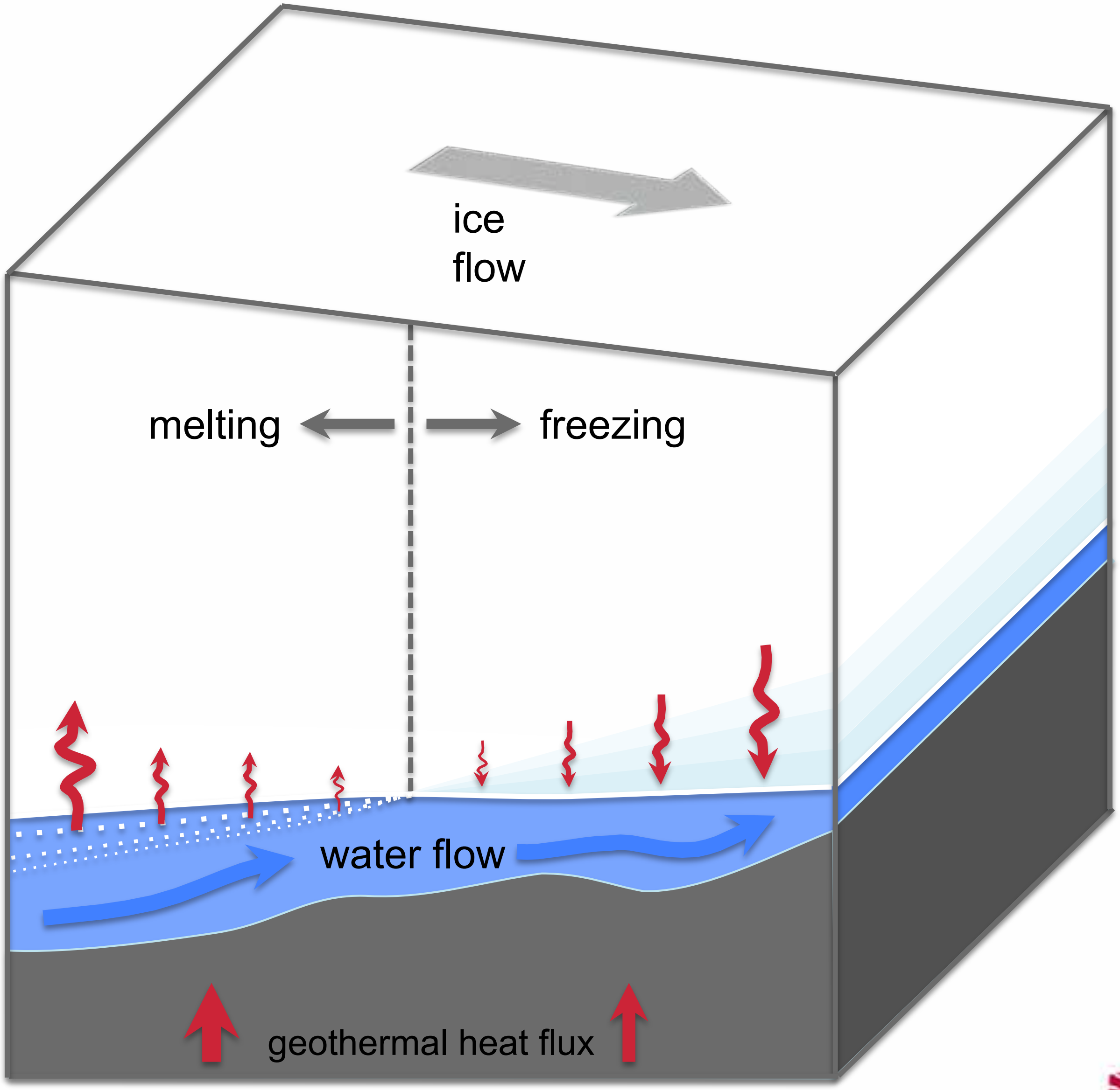
The hydrological system



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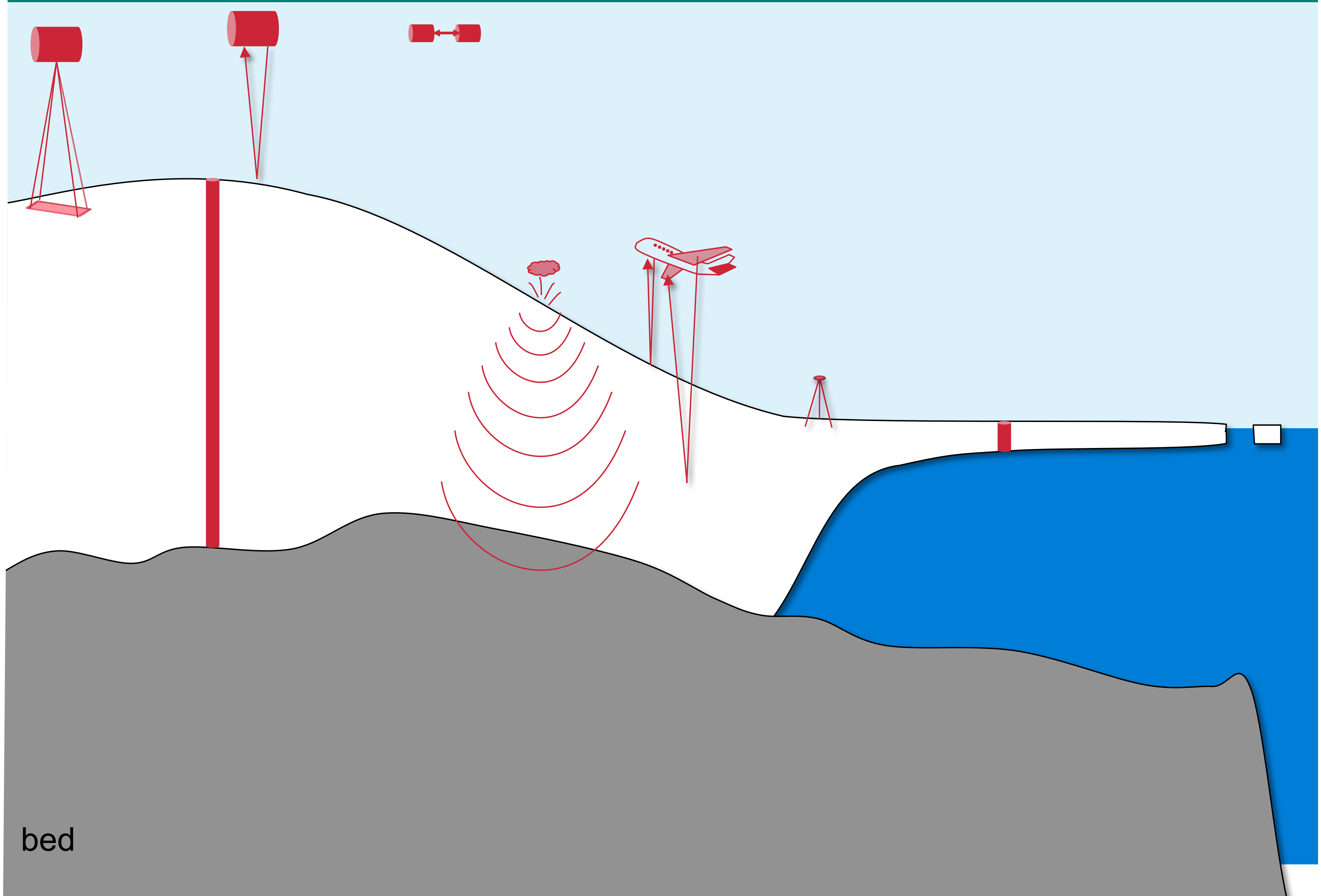
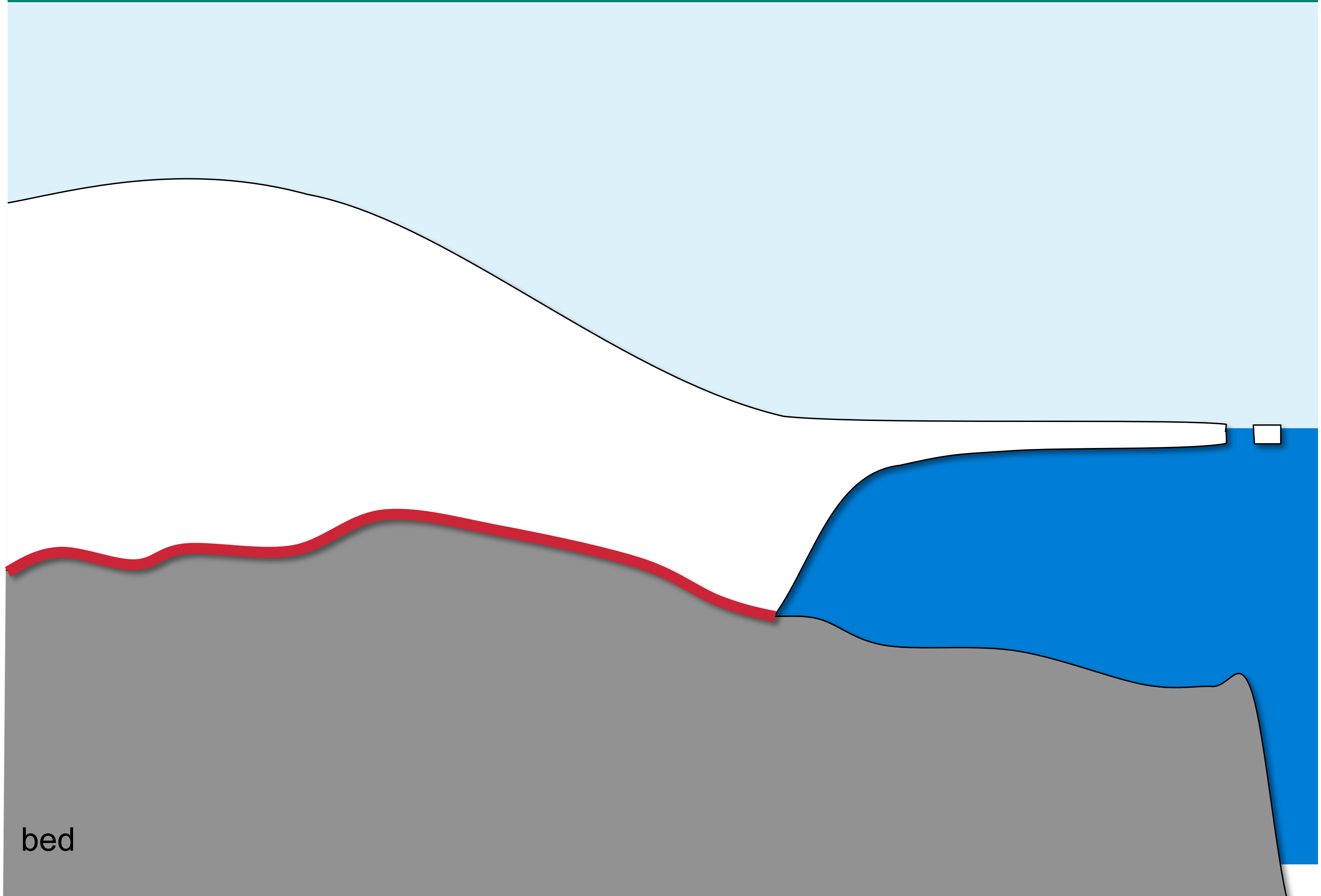
Observational Methods



Observational Methods



Physics of polycrystalline ice

Balance equations:

- **Mass balance**
- **Momentum balance**
- **Energy balance (kin+internal)**

Constitutive equations

- **Incompressible non-Newtonian fluid - Glen‘s flow law**

$$D = EA(T,W)f(\sigma)\,t^{\mathrm{D}}, \quad \text{with } f(\sigma) = \sigma^{n-1}, \; n = 3,$$

empirical, Glen / Steinemann 1955/58

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Mathematical description

Balance equation of ....

Mass (incompressible) $\nabla \boldsymbol{u} = 0$

Momentum $\nabla \sigma = \rho_{ice} \boldsymbol{g}$

Energy $\rho c_p \frac{dT}{dt} = \nabla \cdot (k \nabla T) + 4 \mu d_e^2$

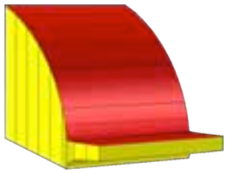
Constitutive equation $\mu(T, p, d_e) = \frac{1}{2}[EA(T,p)]^{-1/n} d_e^{(1-n)/n}$

with $\sigma = t^D - p\boldsymbol{I}$ $D = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$

$t^D = 2\mu D$ $d_e = \sqrt{\frac{1}{2} \mathrm{tr} D^2}$

Boundary conditions

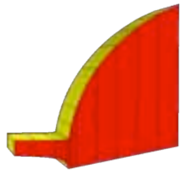
Surface



$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = 0$$

stress-free

Lateral margins and ice divide



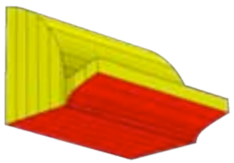
$$(\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{t} = 0$$
$$\boldsymbol{u} \cdot \boldsymbol{n} = 0$$

symmetry, free slip

Base

Floating

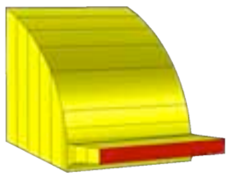
$$(\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{n} = -\rho_{sw} g(-z_b)$$
$$(\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{t} = 0$$



Grounded

$$(\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{n} = -\rho_i g H$$
$$(\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{t} = C|\boldsymbol{u_b}|^{m-1}\boldsymbol{u_b}$$
$$\boldsymbol{u} \cdot \boldsymbol{n} = 0$$

Calving front



$$(\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{n} = -\rho_{sw} g(-z)$$
$$\text{for} \quad (z < 0)$$

water pressure

Temporal evolution

$$\frac{\partial b}{\partial t} + v_x \frac{\partial b}{\partial x} + v_y \frac{\partial b}{\partial y} - v_z = N_\mathrm{b} a_\mathrm{b}^\perp$$

$$\frac{\partial h}{\partial t} + v_x \frac{\partial h}{\partial x} + v_y \frac{\partial h}{\partial y} - v_z = N_\mathrm{s} a_\mathrm{s}^\perp$$

$$\frac{\partial H}{\partial t} = -\mathrm{div}\,\mathbf{Q} + a_\mathrm{s} - a_\mathrm{b}$$

Sprachebenen